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The Perturbation Of Material Density In f(R) Modified Gravity Of Polynomial Exponential Form

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Abstract

In this paper, we investigate the linear perturbation of material density of universe in f(R) modified gravity of polynomial exponential form on the scale of distance below the cosmic horizon (sub-horizon). The results show that the model for the evolutionary aspects of universe is slightly different from that in the standard cosmological model of ΛCDM. They can be used to show the difference of this modified gravitational model with the standard cosmological model of ΛCMD and other cosmological models. We also investigate the ration Ψ/Φ and G_{eff}/G_N in the model and show that they are within allowable limits of experiments.

1 Introduction

Since 1998, scientists have confirmed that our universe is actually accelerating from many observational data [1, 2, 3, 4]. In order to explain this accelerated expansion, scientists have come up with several approaches listed in two main classes [5, 6].

In the first class, this expansion is caused by scalar fields such as the quintessence model [7] and k-essence [8]. In the second class, the expansion is caused by the change of Einstein’s gravity on cosmological distances as f(R) modified gravities [9], tensor - scalar models [10] and brane world models [11].

In this paper, we investigate the evolution of material perturbation□m on cosmic time in a small class of f(R)modified gravity, which called f(R)modified gravity of exponential - polynomial form. The results suggest that these evolutionary aspects can be used to show the difference of this modified gravitational model with the standard cosmological model of ΛCMD and other cosmological models.

In Einstein's gravity, the linear perturbation over sub-horizon distances satisfies the following equation:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \quad .(1)$$

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Here H is the Hubble parameter, G is the Newtonian gravitational constant, $\rho_m$is the energy density of non-relativistic matter, and the dot signs the derivative with respective to cosmic time t. In the matter dominated era, the solution for the growth mode $\delta_m \propto a \propto t^{2/3}$ leads to the formation of large scale structures of universe. In modified gravitational models, the perturbed levels are different due to the correction of the gravitational constant as well as the change in the background. In f(R) modified gravities, in particular, there has been some recent works on the evolution of perturbed density in the matter dominated era and began the vacuum dominated era[12].

This article is structured as follows: in section 2, we present cosmological perturbation in standard universe model ΛCDM; in Section 3, we consider perturbations of material density in f(R)modified gravities; in section 4, we investigate the evolution of material perturbation in f(R) modified gravity of polynomial exponential form; section 6 is the conclusion of the paper.

2 Cosmological Perturbation In The Standard Cosmological Model Λcdm

The metric of perturbed FLRW in longitudinal gauge is

$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Psi)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) . \quad (2)$$

In the matter dominated era, the non-zero components of energy - momentum tensor are

$$T_0^0 = -\rho_m - \delta\rho_m \; , T_i^0 = -\rho_m \partial_i v \; , \quad (3)$$

Here ρm and δρm are the material density and its fluctuation, respectively. υ is the velocity of scalar perturbation.

From the Einstein equations, we can derive the differential equation for the density of comoving as follows

$$\delta = \frac{\delta\rho_m}{\rho_m} + 3Hav \; , \quad (4)$$

Fourier transformation of density perturbation is given by

$$\delta_k(t) = \int \frac{d^3x}{(2\pi)^{3/2}} \delta(t,\vec{x}) e^{i\vec{k}\vec{x}} \; , \quad (5)$$

here k is the sign of a comoving wavenumber, in the following sections, δ k(t) is written as δ for simplicity. The evolution of δ in Fourier space is given by the equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m \delta = 0 \; , (6)$$

in matter dominated era, the equation becomes

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0 \; . (7)$$

The equation has two independent solutions $t^{2/3}$ and $^{-1}$,we only note the growth mode $^{t2/3}$

$$\delta_k(t) = \delta_{0k} (\frac{t}{t_0})^{2/3} \; , \quad (8)$$

Here δ_{0k} is an initial value, it is$\delta_k(t_0)$.

3 The Perturbation Of Material Density In F(R) Modified Gravity

We write the action in the following form [13]

$$S=\int d^4x\sqrt{-g}[\frac{1}{16\pi G}f(R)+L_m], \quad (9)$$

here G is Newtonian constant and Lm is material Lagrangian.

When f (R) = R − 2Λ , we return the standard cosmological model of ΛCDM. Below we consider Lagrangian without cosmological constant and f (R) vanishes when R = 0. In f(R) gravity, modified Einstein's equations are as follows:

$$F R_{\mu\nu}-\frac{1}{2}fg_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)F=8\pi GT_{\mu\nu}, \quad (10)$$

Here

$$F\equiv\frac{df}{dR}. \quad (11)$$

or like – dust matter, the background equations have the form

$$3FH^2=\frac{1}{2}(FR-f)-3H\dot{F}+8\pi G\rho_m, \quad (12)$$

$$-2F\dot{H}=\ddot{F}-H\dot{F}+8\pi G\rho_m, \quad (13)$$

$$\dot{\rho}_m+3H\rho_m=0. \quad (14)$$

Here the dot directs the derivative with respective to the cosmic time. At the end of the universe, when the energy density of the effective dark energy and dust matter are comparable, we have

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}(\rho_m+\rho_{de}), \quad (15)$$

$$2\frac{\ddot{a}}{a}+(\frac{\dot{a}}{a})^2=-8\pi G\omega\rho_{de} \quad (16)$$

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\frac{\rho_m}{\Omega_m}. \quad (17)$$

From the equation (15), (16) and (17), we have

$$\frac{\ddot{a}a}{\dot{a}^2}=-\frac{1}{2}-\frac{3}{2}\omega(1-\Omega_m). \quad (18)$$

When f(R) gravity deviates few from Einstein's gravity, i.e., f (R) ≅ R and F ≅1, these equations lead to $a(t)=a_o(t/t_o)^{2/3}$, in matter dominated era. The differential equation describes the density perturbation in the sub horizon region is[19]

$$\ddot{\delta}+2H\dot{\delta}-4\pi G_{eff}\rho_m\delta=0, \quad (19)$$

here

$$G_{eff} = \frac{1}{1+F_R} \frac{1+4\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}}{1+3\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}} G_N \quad (20)$$

with $F_R \equiv \frac{d(\mathrm{f}-\mathrm{R})}{dR}$ and $F_{RR} \equiv \frac{d^2(f-R)}{dR^2}$

4 Cosmological Perturbation In F(R) Modified Gravity Of Polynomial Exponential Form

In this section we research the evolution of material perturbation in f(R) modified gravity of polynomial exponential form. The action f(R) has the form[15, 16, 17]

$$f(R) = R + a + \frac{\alpha}{R^m}\left(1 + bR^2 + cR^3\right)e^{-\beta R^n}, \quad (21)$$

with $\alpha > 0, \beta > 0; m = n = 1; a = -2\Lambda; b = c = 1$

Expanding approximately f(R) and with hypothesis that β R <<1

We have

$$e^{-\beta R} = 1 - \beta R, \quad (22)$$

$$F_R = \alpha\left[-3\beta R^2 - \frac{1}{R^2} - 2(\beta-1)R + 1\right], \quad (23)$$

$$F_{RR} = 2\alpha\left[-\beta + \frac{1}{R^3} - 3\beta R + 1\right]. \quad (24)$$

In the case, the effective gravitational constant has the form as follows

$$G_{eff} = \frac{1}{1+F_R} \frac{1+4\frac{k^2}{a^2 R} m}{1+3\frac{k^2}{a^2 R} m} G, \quad (25)$$

with

$$m = \frac{RF_{RR}}{F_R + 1}, \quad (26)$$

$$m = \frac{RF_{RR}}{1+F_R} = \frac{2\alpha\left[3\beta R^4 + (\beta-1)R^3 - 1\right]}{\alpha + 3\alpha\beta R^4 + 2\alpha(\beta-1)R^3 - (\alpha+1)R^2}. \quad (27)$$

4.1 The case of $\frac{k^2}{a^2 R} m \gg 1$

In this case, we have $G_{eff} \approx \frac{1}{1+F_R} G.\frac{4}{3}$, the model returns to the Brans – Dicke model with $\omega_{BD} = 0$ [18]. In fact, the case $\frac{k^2}{a^2 R} m >> 1$ can be done in sub-horizon approximate as long as m is not very smaller unit.

The equation (19) can be written in the following form after change the variable

$$\delta'' + \left(\frac{1}{2} - \frac{3}{2}\omega_{eff}\right)\delta' - \frac{3}{2}\left[\frac{1+4\frac{k^2}{a^2 R} m}{1+3\frac{k^2}{a^2 R} m} G\right]\Omega_m \delta = 0 , \quad (28)$$

here

$$\mathrm{N} = \ln a; \ ' = \frac{d}{dN} = \frac{1}{H} dt , \quad (29)$$

and

$$\omega_{eff} = -1 - \frac{2}{3}\frac{H'}{H} \ ; \ \Omega_m = \frac{\rho_m}{3FH^2} \ ; \ \xi = \frac{k^2}{a^2 R} m \ ; \ m = \frac{RF_{RR}}{1+F_R} \quad (30)$$

With the condition $\frac{k^2}{a^2 R} m >> 1$, the equation (28) becomes

$$\delta'' + \left(\frac{1}{2} - \frac{3}{2}\omega_{eff}\right)\delta' - 2G\Omega_m \delta = 0 . \quad (31)$$

In the matter dominated era

$$\omega_{eff} = -\frac{m}{1+m} \text{ and } \Omega_m = 1 - \frac{m(7+10m)}{2(1+m)^2} . \quad (32)$$

We choose the solution in the following form

$$\delta = c_+ a^{n_+} + c_- a^{n_-} . \quad (33)$$

Solving (31), we have the solution

$$n_\pm = \pm \frac{\sqrt{-(m+1)^2[16(8m^2+3m-2)-(4m+1)^2]} + 4m^2 + 5m + 1}{4(m+1)^2} \quad (34)$$

A solution of the growth mode is $\delta_m = c_+ a^{n_+}$

Here

$$n_+ = \frac{\sqrt{-(m+1)^2[16(8m^2+3m-2)-(4m+1)^2]} + 4m^2 + 5m + 1}{4(m+1)^2} . \quad (35)$$

In matter dominated era, the scale factor a(t)depends on cosmic time t in the form

$$a(t) \propto t^{2/3} , \quad (36)$$

So the growth mode of material perturbation depends on cosmic time in the form

$$\delta = c_+ t^{\frac{2}{3}n_+} \quad (37)$$

With n+ like in the expression (35).

Figure 1 shows the growth mode of cosmological perturbation on the scale factor a and cosmic red – shift parameter $z = a_{o/}\ a - 1$

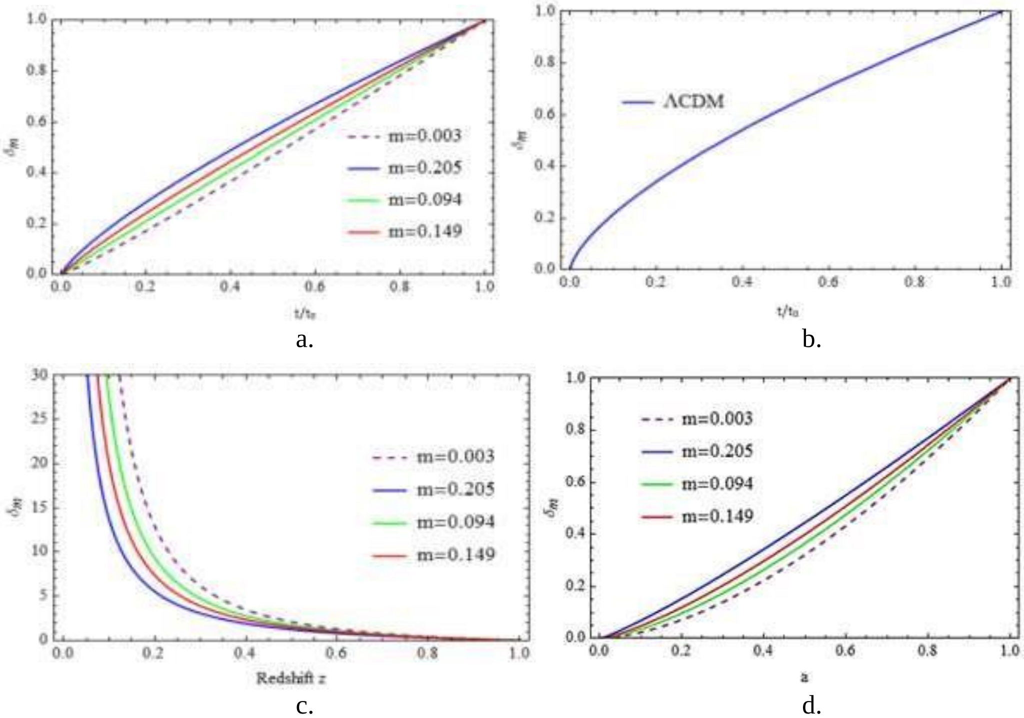

Figure1. Graphs with different values of m, they are compared to that in standard cosmological modelΛCDM

In figure.1a, as m approaches to zero, the graph is significantly different from that in the standard cosmological model; figure 1b is the graph in the standard cosmological model

ΛCDM; in figure 1c, the perturbation δ as a function of redshift z; in figure 1d, perturbation δ as a function of scale factor a(t).

4.2 The case $\frac{k^2}{a^2 R} m \ll 1$

This condition is satisfied on distance scales that large structures of universe were made. In this case, equation (28) becomes

$$\delta'' + \left(\frac{1}{2} - \frac{3}{2}\omega_{eff}\right)\delta' - \frac{3}{2}\Omega_m G\delta = 0. \quad (38)$$

Solving equation (38), we obtain

$$\delta = c_+ a^{\frac{1}{4}\left[-\sqrt{\frac{(4m+1)^2 - 12G(8m^2+3m-2)}{(m+1)^2}} - \frac{3m}{m+1} - 1\right]} + c\_a^{\frac{1}{4}\left[\sqrt{\frac{(4m+1)^2 - 12G(8m^2+3m-2)}{(m+1)^2}} - \frac{3m}{m+1} - 1\right]}. \quad (39)$$

The solution of the growth mode is

$$\delta = a^{\frac{1}{4}\left[\sqrt{\frac{(4m+1)^2-12G(8m^2+3m-2)}{(m+1)^2}}-\frac{3m}{m+1}-1\right]}, \quad (40)$$

or in the other form, it depends on cosmic time as follows

$$\delta = t^{\frac{1}{6}\left[\sqrt{\frac{(4m+1)^2-12G(8m^2+3m-2)}{(m+1)^2}}-\frac{3m}{m+1}-1\right]}. \quad (41)$$

When $m=0 \rightarrow \delta \propto a \propto t^{2/3}$.

Figure 2 shows cosmological perturbation in the growth mode

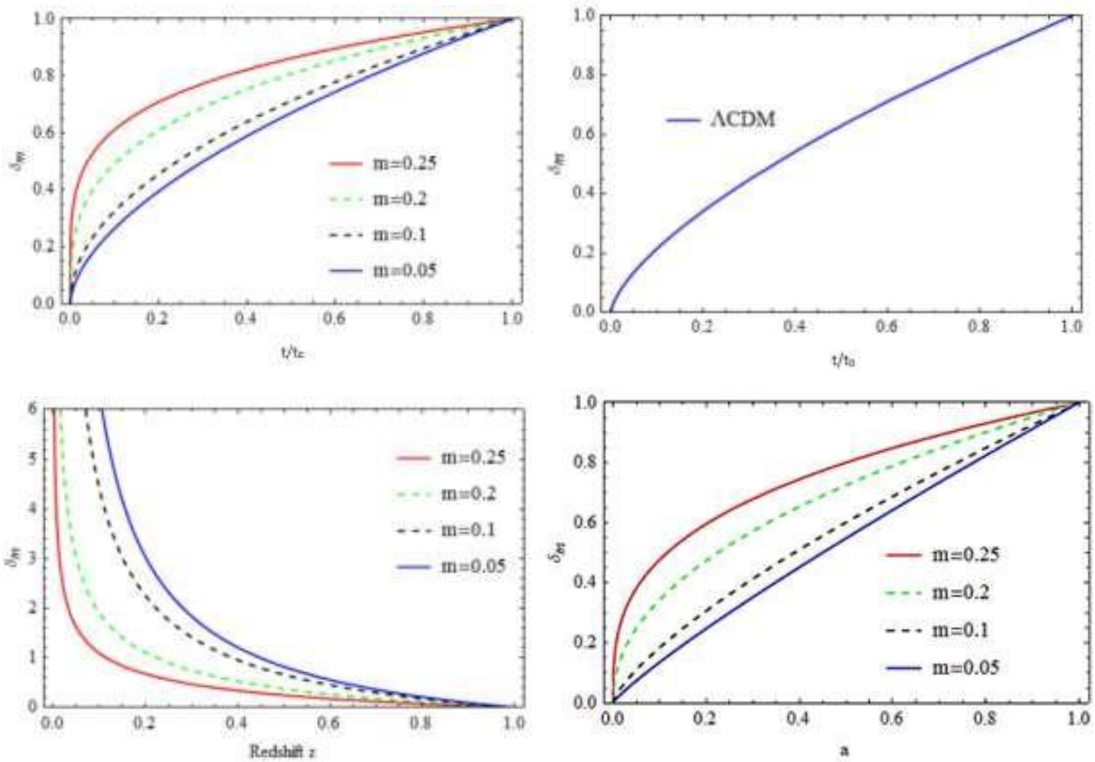

Figure 2.Graphs with different values of m, they are compared to that in standard cosmological modelΛCDM.

In figure.2a, as m approaches to zero, the graph is the same with that in the standard cosmological model, when large m, graph is significantly different from that in it; figure 2b is the graph in the standard cosmologicalΛCDM;in figure 2c, the perturbation δ as a function of redshift z; in figure.2d, perturbation δ as a function of scale factor a.

5 The Evaluation Of The Ratio Ψ/ Φ

In this section we will re-evaluate the ratio of the scalar functions Ψ/Φ in this model and compare it to the experimental constraints. We have [19],

$$\frac{\Psi}{\Phi}=\frac{1+4\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}}{1+2\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}}, \quad (42)$$

Here k = 0.01hMpc^{-1} ~ 10-^{25}m^{-1} ; h = 0.72±0.08,R~ 10^{-30}m^{-2}, $\alpha = 9\times10^{-75}$[16].

In the present time, the scale factor a = 1, so we have in f(R) modified gravity of polynomial exponential form

$$F_R \propto -\frac{\alpha}{R^2};\mathrm{F}_{RR}\propto\frac{2\alpha}{R^3};\frac{F_{RR}}{1+F_R}\propto 2\alpha R^{-3}, \quad (43)$$

Therefore Ψ/ Φ =1.

In the early stages of the universe, the cosmic curvature is very large, the scale factor a is very small, if $\frac{k^2}{a^2R}m \gg 1$ with $\frac{RF_{RR}}{1+F_R}$, we have

$$\frac{\Psi}{\Phi}\approx 2 \quad (44)$$

The experimental data when k = 0.01hMpc$_{-1}$and a = 1[19],

$$1<\frac{\Psi}{\Phi}\le 1.996 \quad (45)$$

The ratio Ψ/ Φ in this model is within the allowable limits of the experiment.

6 The Evaluation Of The Ratio G_{eff} / G_N

In this section, we calculate the ratio of the effective gravitational and the Newtonian constant and compare it with the constraints from experiments.

We have from formula (20),

$$G_{eff}=\frac{1}{1+F_R}\frac{1+4\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}}{1+3\frac{k^2}{a^2}\frac{F_{RR}}{1+F_R}}G_N \quad (46)$$

In the present time [19],R~$10^{-30}m^2$, the scale factor $a=1$, we have

$$F_R=\alpha\left[-3\beta R^2-\frac{1}{R^2}-2(\beta-1)R+1\right]\cong-\frac{\alpha}{R^2} \quad (47)$$

$$F_{RR}=2\alpha\left[-\beta+\frac{1}{R^3}-3\beta R+1\right]\cong\frac{2\alpha}{R^3} \quad (48)$$

With k = 0.01hMpc^{-1} ; h = 0.72±0.08

Here $\alpha;\ \beta$ have the values [16]

$$\alpha=9\times10^{-75};\beta=7.67\times10^{-15} \quad (49)$$

we have
$$\frac{G_{eff}}{G_N} \cong 1 \quad (50)$$

In the early stages of the universe, the cosmic curvature is very large, the scale factor a is very small. When $\beta R \ll 1 \rightarrow 1 < R \ll 10^{14} m^{-2}$, we have

$$F_R = \alpha\left[-3\beta R^2 - \frac{1}{R^2} - 2(\beta - 1)R + 1\right] \cong -2\alpha R \quad (51)$$

$$F_{RR} = 2\alpha\left[-\beta + \frac{1}{R^3} - 3\beta R + 1\right] \cong -6\alpha\beta R \quad (52)$$

If $\frac{k^2}{a^2 R} m \gg 1$ with $m = \frac{R F_{RR}}{1 + F_R}$, we have

$$\frac{G_{eff}}{G_N} \cong \frac{4}{3} \cong 1.33 \quad (53)$$

The constraints [19] are,

$$1 \le \frac{G_{eff}}{G_N} \le 1.403 \quad (54)$$

The constraint in the recombination epochs [20],

$$\frac{G_{rec}}{G_o} - 1 < 1.9 \times 10^{-3} (95.45\% \text{C.L.})$$
$$\frac{G_{rec}}{G_o} - 1 < 5.5 \times 10^{-3} (99.99\% \text{C.L.}) \quad (55)$$

We see that the G_{eff} /G_N ratio changes according to the evolution of the universe, depending on the scale factor a, which has the largest deviation from unit at the early stage of the universe, but it almost coincides with unit in the current stage from this model.

7 Conclusion

In this article, we have used analytical method and numerical calculations to investigate the evolutional aspects of perturbation of material density in f(R) modified gravity of exponential – polynomial form. The evolutionary aspects in this model are compared to that in the standard cosmological model of ΛCDM. The results show a significant difference in the evolutionary aspect of perturbation in this model versus the standard model and can be used to distinguish it from the standard model and other models. The ratios Ψ/ Φ and G_{eff} /G_N in this model are also within the allowable limits of experimental constraints.

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