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# Mathematical-Logistic Model of Integrated Production Structure of Food Production

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**Abstract.** The article is devoted to the formation of a technological-logistic model of the integrated structure of food production. The main goal of corporate structure management is the integration of all its constituent units for the fulfillment of the mission, which ensures achievement of the set goals. The main purpose of modeling is to show how the intermediate links-enterprises are logically formed the target object. A mathematical formulation of the problem of choosing optimal capacities and rational location of enterprises, as well as minimum costs for transportation of raw materials, is proposed. A complex mathematical model for planning the production of agricultural raw materials and processing it into ready-made food products in the system "agricultural sector - provision / primary processing - food industry enterprises" was formed. Model of the logistic organization of integrated food production are based on the principles of rational organization of the technological chain and are characterized by: complexity, universality, differentiation of the approach; specialization. The developed mathematical model allow planning and programming of the development processes of the integrated food production system, assessing the impact of changes in the parameters of the system, and adjusting plans. With the help of Statgraphics, Statistica, Excel software and having as a basis an array of enterprise data, it is possible to plan and program the development processes of an integrated food production system, assess the impact of changes in system parameters, make adjustments to plans. The model make it possible to specify the technological complex of work and the need for raw materials, provide an opportunity to establish boundaries between complexes of works of individual companies and, in general, the responsibility of the entire corporate structure.

**Keywords:** logistic model, food industry, agrarian sector, integrated production, economic-mathematical modeling

## 1. Introduction

The food industry is one of the few branches of the Ukrainian economy, which is in the stage of steady development. However, the raw material base of food production – the agricultural sector of Ukraine today is in a crisis condition, characterized by a decline in the material and technical base, high costs of production, shortage of working capital.

Measures to implement integrated development strategy should be scientifically sound and rely on mathematical models of processes that will take place in a new production system.

Technological peculiarities of the production of agrarian products, as well as the complexity of the processes of its harvesting, accumulation and, if necessary, primary processing, processing into finished products, make a relevant mathematical modeling of the planning of production of raw materials, taking into account possible changes in production volumes. The purpose of the article is to study the technological and logistical side of the rational organization of the integrated complex of enterprises "agriculture - procurement organizations - enterprises of the food industry". The objectives of the paper are: the identification of production, economic and transport factors of the interaction of production in the technological chain of food production and the mathematical formulation of the task of choosing the optimal capacities and rational allocation of the integrated production complex enterprises, as well as the minimum costs for the transportation of raw materials.

## 2. Related Works

It should be noted that the issue of the placement of specialized agricultural enterprises and the transportation of products produced by agrarian enterprises has been given attention in the literature since the 60-s of the last century [1].

The issue of solving problems of optimization of processes in agrarian and processing sectors of the economy of Ukraine and today are at the center of attention.

So in the works of Y. Brodsky [2] and S. Nakonechny [3] shows the economic and mathematical functioning of agrarian enterprises, in particular models of production structure, innovation processes, technical and economic processes. These models have a high level of reliability, however, they cover only the primary link in the technological and logistic chain of food production.

Separately allocated works related to the simulation and optimization of logistics processes in the industry, such as scientific work Y. Borbot [4]. Problems of find optimal solutions in the industrial production and logistics system have been analyzed in articles of modern scientists, like A. Gola [5]. But we can't say about existing of universal optimization model. Individual model of optimization of food production and logistics system of particular kind of food product must consider all set of

specific features of the sector.

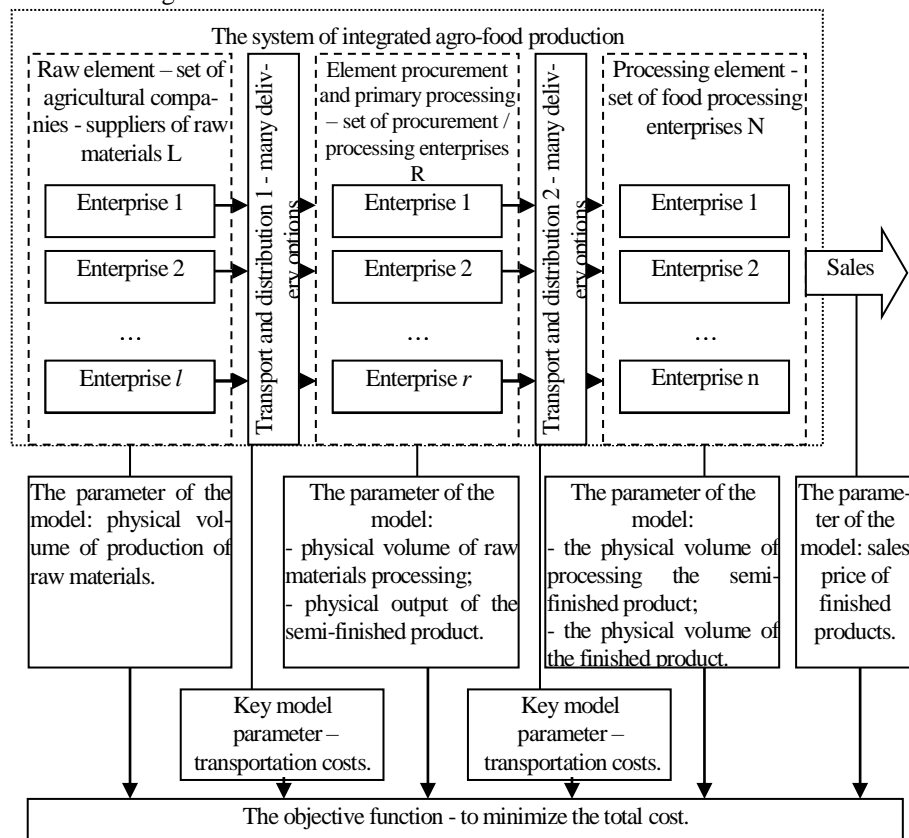
It should be emphasized that market requirements today require the formation of a complex integrated system of agro-industrial food production, which will enable to realize the reserves of competitiveness of the industry. Consequently, it is necessary to approach the modeling of technological and logistic processes in a complex way. The necessity of mathematical modeling of the planning of the enterprises of the food industry and its raw material base is caused first of all by the possibility in this case of more rational use of available resources and optimization of commodity-cash flows [6, 7].

Taking into account that the question of modeling the optimal development of the food production complex remains open, we will try to propose an own view on its solution in this paper.

### 3. Research Methods

The tasks of logistic management of commodity flows in order to minimize costs and maximize profits in the food production system is a complex methodological task, as the technological chain of food production involves a large number of actors that enter into interaction. Consequently, it is necessary to take into account the whole set of participants in production and to coordinate their numerous interactions, taking into account their functional features of their organization and the technology of the work.

To solve this problem we used the instrumental apparatus of mathematical modeling, namely - setting up an optimization problem Z-type with a system of constraints, which allows the most adequately describe the investigated technological and logistic system. During the study was developed the model of optimization of capacities of production units and minimization of expenses for transportation of raw materials. Limitations of the model are the volumes of raw material production, the quantities of raw materials delivered between the stages of the technological process and the volume of raw materials transported between all the links of the production-technological chain. Method of formation and structural composition of the model are shown on the fig. 1.



**Fig. 1.** Component of the model for optimizing production and logistics chains of an integrated agrarian-food production structure

The system should strive to get close to the ideal state of operation, which will ensure the economy of resources and maximize returns.

For the practical use of this model, the formation of an appropriate information environment is required by monitoring and accumulating the statistical base of parameters that characterize the use of resources, costs, production and logistics and transport and logistics flows.

#### 4. The Proposed Optimization Model

In the complex of integrated production, a number of small agricultural enterprises, several harvesting organizations and / or primary processing enterprises, one or several enterprises of the food industry are connected. All of them are, as a rule, geographically located in one region or neighboring regions of Ukraine - in the zone of growing of raw materials. Since each enterprise of the next stage is a consumer of raw materials or semi-finished products produced by enterprises of the previous stages, the desired quantities can be represented in homogeneous units of measurement by means of conversion into a single conditional product.

The mathematical model of the optimal development of the food production complex consists of the objective function  $F(x)$ , which expresses the general minimum expected costs of growing the raw material, its harvesting and primary processing and transportation of products:

$$F(x) = \sum_{j=1}^n \sum_{k=1}^l C_{jk} x_{jk} + \sum_{j=1}^n \sum_{k=1}^l C^*_{jk} x^*_{jk} + \sum_{j=1}^n \sum_{k=1}^l \sum_{i=1}^r S_{jki} x_{jki} + \sum_{j=1}^n \sum_{k=1}^l \sum_{i=1}^r C_{jki} x_{jki} + \sum_{j=1}^n \sum_{i=1}^r \sum_{m=1}^p S_{jim} x_{jim} \rightarrow \min, (6)$$

where:  $k$  is one of the plurality (1,2,3, ...,  $k$ , ...,  $l$ ) of agricultural enterprises producing agrarian raw materials and supplying it to procurement organizations and / or primary processing enterprises;  $j$  is one of the plural (1,2,3, ...,  $j$ , ...,  $n$ ) type of raw material produced by the  $k$ -th agricultural enterprise;  $i$  is one of the plurality (1,2,3, ...,  $i$ , ...,  $r$ ) of procurement organizations and / or primary processing enterprises;  $m$  is one of the plural (1,2,3, ...,  $m$ , ...,  $p$ ) enterprises of the food industry, which produces the final product;  $\tau$  - one of the plural (1,2,3, ...,  $\tau$ , ...,  $t$ ) years of the planned period of development of food production;  $C_{jk}$  - production costs of the unit of raw material of the  $j$ -th species in the  $k$ -th agricultural enterprise;  $C^*_{jk}$  - costs related to the expansion of the sown area / livestock to obtain an additional unit of raw material of the  $j$ -th species in the  $k$ -th agricultural enterprise;  $C_{jki}$  - expenses for processing of the unit of volume of raw material of the  $j$  type, received from the  $k$ -th agricultural enterprise at the  $i$ -th enterprise of primary processing;  $S_{jki}$  - transportation costs per unit of raw material of  $j$ -th type from  $k$ -th agricultural enterprise to  $i$ -th enterprise of primary processing and / or procurement organization;  $S_{jim}$  - transportation costs per unit of raw material  $j$ -th type from the  $i$ -th primary processing enterprise and / or procurement organization to the  $m$ -e enterprise of the food industry;  $v_j$  - the largest volume of raw materials of the  $j$ -th type, which should be developed by all 1

agricultural enterprises;  $V_j$  - the largest volume of  $j$ -th type raw material, which can be taken for processing from all  $l$  agricultural enterprises all  $r$  of the enterprise of primary processing;  $W_j$  - the largest volume of  $j$ -th type raw material, which can be taken for processing by all  $r$  of procurement organizations and / or enterprises of primary processing all  $p$  enterprises of the food industry;  $x_{jk}$  - the desired amount of raw material of the  $j$ -th species, which should be developed in the  $k$ -th agricultural enterprise;  $x^*_{jk}$  - the required additional amount of  $j$ -th type raw material to be produced in the  $k$ -th agricultural enterprise;  $x_{jki}$  is the desired amount of  $j$ -th type raw material delivered from the  $k$ -th agricultural enterprise to the  $i$ -th primary processing enterprise and / or procurement organization;  $x_{jim}$  - the required volume of  $j$ -th type raw material, delivered from the  $i$ -th enterprise of primary processing and / or procurement organization to the  $m$ -th enterprise of the food-processing industry;  $Go_{jk}$  - quantity (in units of measurement) of  $j$ -th type raw material at  $k$ -th agricultural enterprise;  $Gn_{ji}$  - the need (in units of measurement) of  $j$ -th type raw material at the  $i$ -th enterprise of primary processing and / or stockpiling organization;  $Go_{ji}$  - the quantity (in units of measurement) of  $j$ -th type raw material at the  $i$ -th enterprise of primary processing and / or procurement organization;  $Gn_{jm}$  - demand (in units of measurement) of raw material of the  $j$ -th type at the  $m$ -th enterprise of the food-processing industry/

Restrictions of model:

1. The gross output of  $j$ -type raw material by all  $l$  agricultural enterprises must be agreed in advance with all enterprises of primary processing and / or procurement organizations:

$$\sum_{k=1}^l (x_{jk} + x^*_{jk}) \leq V_j; \quad (7)$$

2. The total volume of deliveries of raw materials of the  $j$ -th type to all  $l$  agricultural enterprises should not exaggerate the possibilities of its processing by all enterprises of primary processing:

$$\sum_{k=1}^l \sum_{i=1}^r x_{jki} \leq V_j; \quad (8)$$

3. The total volume of supplies of  $j$ -type raw materials by all enterprises of primary processing and / or procurement organizations should not exaggerate the possibilities for its processing by all enterprises of the food industry:

$$\sum_{i=1}^r \sum_{m=1}^p x_{jim} \leq W_j; \quad (9)$$

In this case, to perform compatibility of the conditions of the problem, it is necessary that there are fair inequalities:

$$v_j \leq V_j \leq W_j. \quad (10)$$

In the set task the criterion of optimality is taken the minimum of production and transport costs. With the closed model of the transport task to the specified restrictions, the following is added:

4. The gross volume of  $j$ -type consignments shipped from all  $l$  agricultural enterprises should correspond to the total demand for these cargoes at the destination points of each of the  $r$  enterprises of primary processing and / or procurement

organizations:

$$\sum_{j=1}^n \sum_{i=1}^r x_{jki} = G^o_{jk}; (11) \quad \sum_{j=1}^n \sum_{k=1}^l x_{jki} = G^h_{ji}; (12) \quad \sum_{k=1}^l G^o_{jk} = \sum_{i=1}^r G^h_{ji} (13)$$

The same value should occur when sending materials to the food industry:

$$\sum_{j=1}^n \sum_{m=1}^p x_{jim} = G^o_{ji}; (14) \quad \sum_{j=1}^n \sum_{i=1}^r x_{jim} = G^h_{jm}; (15) \quad \sum_{i=1}^r G^o_{ji} = \sum_{m=1}^p G^h_{jm}. (16)$$

If the problem under consideration is to be formulated in a dynamic statement, then the mathematical model to be derived should be classified in a particular year, which we accept for the first ( $\tau = 1$ ) in the planned period  $t$ . In this case, the target function  $Z(x, \tau)$  takes the form:

$$Z(x, \tau) = \sum_{\tau=1}^t F_{\tau}(x) \rightarrow \min, \quad (17)$$

where  $F_{\tau}(x)$  means that the parameter  $\tau$  is present as an additional index for all parameters and variables of the function  $F(x)$ .

Since the dynamical model implies the need to increase the production plan with each passing year, the relationship must be fulfilled:

$$0 \leq x_{jk\tau} \leq x_{jk(\tau+1)}, \quad 0 \leq x_{jki\tau} \leq x_{jki(\tau+1)}, \quad 0 \leq x_{jim\tau} \leq x_{jim(\tau+1)}. (18)$$

## 5. Results and discussion

The offered mathematical model allows to carry out planning and programming of processes of development of the new integrated food production system, to estimate influence of changes in system parameters and to make adjustments of plans. The output data for calculating the optimal amount of raw materials for production and processing are given in the table 1.

**Table 1.** The output data for calculating the optimal amount of raw materials

The raw material base								
Farms	Area under crops, ha	Crop capacity, t/ha	Available amount of raw materials, t	Distribution of raw materials ( $G_{0jk}$ ), t		Unit production costs ( $C_{jk}$ ), uah/t	Transportation costs ( $S_{jki}$ ), UAH/t	
				Kherson	Nash Product		Kherson	Nash Product
1. Chaika	10	70	700	700	-	7000	30	35
2. Lotos	27	60	1620	800	-	6000	25	30
3. Ukraine	20	55	1100	500	-	5000	20	25
4. Druhba	50	40	2000	-	2000	5000	30	24
5. Ahrkom	35	30	1050	-	1000	4000	20	15
Primary processing								
Processing enterprises	Need for raw materials ( $G_{njk}$ ), t		Output products, t	Unit production costs ( $C_{ji}$ ), UAH/t		Costs of transportation to the food business ( $S_{jim}$ ), UAH/t		

1. Kherson	2000	350	11000	15
2. Nash product	3000	500	10000	25
Production of finished products				
Enterprises	Volume of consumed raw materials ( $G_{nijm}$ ), t	Unit production costs ( $C_{jim}$ ), UAH/t	Price of the finished product, UAH/t	Total cost $F(x)$ , UAH
1. Pani Kristina	850	5000	40000	39431750

A real system of production of tomato raw materials (tomatoes → tomato paste → ketchup), localized on the territory of Belozersky district of the Kherson region, was selected to test the model. The system consists of three production steps. The first stage is the raw material base, which is represented by farms (F) “Chaika”, “Lotos”, “Ukraine”, “Druzhba” and “Ahrokom”, which grow tomatoes. The second stage is the primary processing, which is represented by processing enterprises LLC Fruit and Vegetable Plant “Kherson” and PE “Nash Product”, which produce tomato paste. The third stage is the enterprise of the food industry of PICF "Pani Kristina", which produces the final product - ketchup under the trademark "Holiday".

The calculation of the model was made using the SAS Model Manager software. Results of optimization of the model are presented in the table 2.

**Table 2.** Calculating the optimal amount of raw materials

The raw material base								
Farms	Area under crops, ha	Crop capacity, t/ha	Available amount of raw materials, t	Distribution of raw materials ( $G_{0jk}$ ), t		Unit production costs ( $C_{jk}$ ), uah/t	Transportation costs ( $S_{jki}$ ), UAH /t	
				Kherson	Nash Product		Kherson	Nash Product
1. Chaika	10	70	700	-	-	7000	30	35
2. Lotos	27	60	1620	850	-	6000	25	30
3. Ukraine	20	55	1100	1100	-	5000	20	25
4. Druzhba	50	40	2000	50	1950	5000	30	24
5. Ahrokom	35	30	1050	-	1050	4000	20	15
Primary processing								
Processing enterprises	Need for raw materials ( $G_{njk}$ ), t	Output products, t	Unit production costs ( $C_{ji}$ ), UAH/t	Costs of transportation to the food business ( $S_{jim}$ ), UAH/t				
1. Kherson	2000	350	11000	15				
2. Nash Product	3000	500	10000	25				
Production of finished products								
Enterprises	Volume of consumed raw materials ( $G_{nijm}$ ), t	Unit production costs ( $C_{jim}$ ), UAH/t	Price of the finished product, UAH/t	Total cost $F(x)$ , UAH				
1. Pani Kristina	850	5000	40000	38025050				

According to the results of the calculation, optimal volumes of production and supply of raw materials and semi-finished products in the technological chain of



production and processing of tomatoes were determined. In the basic, actually existing (non-optimal) version of production, the total amount of expenses is 39431750 UAH, the optimal amount of expenditures is  $F(x) = 38025050$ . The obtained data allow to reduce expenses for production and transportation of products, increase production efficiency. In particular, the cost saving is 1406700 UAH.

It is possible to recommend "Chaika" to refuse to produce raw materials in favor of other types of products, due to economic impracticability. It is recommended to reduce volumes of tomato crop area for Lotus, and it is advisable to revise programs for the supply of raw materials to processing plants. In particular, the part of raw materials from "Druzhba" should be sent to the processing plant LLC Fruit and Vegetable Complex "Kherson".

## 6. Conclusions and Outlook

The construction of a cost management system in integrated food production should be based on the principle of feedback, that is, on the needs of food industry enterprises, which are conditioned by the market conditions. The resource management cycle, like the whole system of control of the technological chain, should cover all stages of product creation.

The offered model of the technological-logistic integrated structure of food production should be used in practice in the activity of the enterprise of the food sector.

With the help of Statgraphics, Statistica, Excel software, and based on the enterprise data array, it is possible to plan and program the development processes of the integrated food production system, to evaluate the impact of changes in the parameters of the system, to make adjustments to the plans.

These programs are most user-friendly for beginners due to the lack of targeting a specific subject area, a wide range of statistical techniques, and a user-friendly interface. They are more accessible to practice and can be used by a wide range of specialists of different profiles.

Using the proposed model will significantly reduce the need for raw materials in the enterprise. In addition, a significant reduction in the likelihood of errors when making managerial decisions.

The presented model allow to specify the technological complex of works and the need for raw materials, provide an opportunity to establish boundaries between the complex of works, for which the producers-executors are responsible and, in general, the responsibility of the entire corporate structure of food production.

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