Licensing AI-Supported Functionality in App Markets under User Base Restriction

Jyh-Hwa Liou, Sheng-Chih Wang, Meng-Ju Lee and Jhih-Hua Jhang-Li

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Introduction

With the rise of generative artificial intelligence (AI), concerns about inappropriate content and data leakage have surfaced, leading to age restrictions and usage bans in some sectors. These restrictions, aimed at safeguarding users, ironically undercut the revenue potential of apps benefiting from generative AI's enhanced functionalities. Technology licensing emerges as a strategic response, enabling AI providers to monetize their intellectual property by licensing it to app developers. This model, however, introduces complexities, including competitive tensions among licensees and the imperative for licensors to adapt their pricing strategies in light of user base restrictions.

App markets, primarily led by Apple and Google, serve as battlegrounds where developers vie for consumer attention, with AI functionalities offering a competitive edge. Yet, the unpredictability associated with AI-generated content poses a challenge, potentially limiting user base and adoption rates. Our study employs a stylized model to explore how AI providers navigate these challenges, focusing on optimal pricing strategies for licensing AI-supported features to app developers. We delve into scenarios involving both independent AI providers and those in vertical integration with app developers, highlighting the nuanced considerations that inform licensing decisions in the face of regulatory constraints and market dynamics.

Our findings underscore the delicate balance AI providers must strike in licensing their technologies, weighing the benefits of widespread adoption against the need to maintain control over their intellectual property and navigate the complex landscape of user base restrictions. This analysis not only sheds light on the strategic dimensions of technology licensing in the digital app market but also highlights the broader implications of regulatory and competitive forces shaping the deployment and commercialization of AI functionalities.

Literature Review

This research is closely related to two streams of literature – technology adoption and licensing. In particular, we focus on the use of royalty licensing in vertical integration literature and review recent development related to user base restriction.

Technology adoption

Consider a supply chain or service chain composed of supply-side (e.g., manufactures or service providers) and demand-side (e.g., consumers or clients). The issue of technology adoption often revolves around the impact of adopting new technology on either supply-side or demand-side (Jin & Li, 2012; Wang et al., 2022). For instance, the adoption of innovative technology often elicits different levels of consumer anxiety. Firms can employ a promotion campaign to help mitigate consumers' disutility of being an early adopter or increase the trust of late adopters by utilizing externality through early adopters. However, though employing a promotion campaign can accelerate the saturation of a market, it could sacrifice the advantage of favorable externality because of less late adopters in the market (Huang et al., 2018).

Licensing and vertical integration

For gaining competitiveness, a retailer can rely on appropriate vertical integration strategies to render required quality of service (Li et al., 2021) or increases its rival's production cost by lowering the number of available suppliers in the market (Hernán González & Kujal, 2012). In addition, vertical integration can even serve as an anti-counterfeiting strategy to help brand firms to deter counterfeiters (Bian et al., 2023). Zhu et al. (2023) conducted a study on vertical integration, wherein a supplier not only determines the wholesale price of its component but also engages in selling its end product within a vertically differentiated market. In their model, the suppler can gain the monopoly profit by rejecting to supply its component or employ the rival's channel to gain additional revenue. However, stronger competitiveness can intensify the price war among firms in a vertically differentiated market. Therefore, Qiu et al. (2023) showed that a low-quality service provider (e.g., an online secondhand platform) itself may not benefit from the adoption of vertical integration when the quality differentiation is not significant.

Prior studies also demonstrated the advantage of employing the rival's channel by licensing a patented technology. In general, when licensors and licensees are competitors in a market, royalty licensing has better capability of coordinating a supply chain than fixed fee licensing, especially when market products show low network effects (Lin & Kulatilaka, 2006; Zhang et al., 2018), the technology gap between licensors and licensees are small (Li & Yanagawa, 2011), or investment funds for developing an innovation are not constrained (Kulatilaka & Lin, 2006). However, the large unit cost gap between licensors and licensees will be detrimental to the benefit of technology licensing (Liu et al., 2022).

Use base restriction

Use base restriction is somehow relevant to the issue of demand uncertainty but differs in the essence of information. The causality of user base restriction is complete information, which can be considered a kind of preventive measure adopted by a platform operator or higher-level executives for avoiding the potential risk caused by AI. On the other hand, demand uncertainty depicts the fact that the future demand cannot be observed in advance because it can fluctuate due to random events such as s climate, trends, and marketing campaigns (Abolghasemi et al., 2020).

In fact, many social networking platforms or internet service providers stemming from their corporate social responsibilities for protecting younger users have regulated the minimum age for accessing their services or products (O'Neill, 2013; Venrick et al., 2023). Nevertheless, prior empirical studies demonstrated that implementing age restrictions for younger audiences can lead to a reduction in revenue for content providers (Lampe & McRae, 2021; Leenders & Eliashberg, 2011). Specifically, age restriction diminishes the appeal of a given product by conveying a signal that online platforms cannot assure consumers of the safety of their services, aligning with the concept of the tainted fruit theory (Bushman, 1998).

Overall, our stylized model is based on a horizontally differentiated market (Chen et al., 2023; Feng et al., 2022; Keskin & Taskin, 2015; Shao, 2020) to investigate the impact of user base restriction on the issue of technology adoption. In this setup, a technology provider uses a royalty fee licensing arrangement as its billing method to license its AI tools for helping service providers increase the value of their apps on an online platform. Moreover, we also study this issue under vertical integration to further understand whether an AI provider will keep its technology exclusive to compete with other service providers in the market.

The Model

Consider an AI provider selling its intelligent engine (known as AI-supported add-on) to two horizontal differentiated app providers on a digital platform. The add-on can seamlessly integrate into apps, offering users an automated content generation feature and creating a highly convenient environment. Both app providers, indexed by $i = A,B$, decide on their app prices and pay a commission fee to the digital platform for each transaction. Let p_i be the app price charged by app provider i and v_i the value a consumer perceives on the app. Following Shy and Stenbacka (2003), app providers are situated at the opposite ends of a Hotelling line with unit length, and consumers uniformly distributed on the line always buy one of the apps. Therefore, the utility of a consumer is expressed as

$$U(\theta) = \begin{cases} v_A - \lambda\theta - p_A & \textit{when buying app A} \\ v_B - \lambda(1-\theta) - p_B & \textit{when buying app B} \end{cases} \quad (1)$$

, where $\theta \in [0,1]$ is the index of a consumer's preference and $\lambda > 0$ measures the disutility a consumer experiences by using a product variety different form her ideal app. Therefore, if the distance between a consumer's preference index θ and the location of app provider i is shorter, the utility of a consumer purchasing the brand sold by app provider i increases. In an extreme case, $\theta = 0$ ($\theta = 1$) represents that the brand sold by app provider A (app provider B) is the consumer's ideal app without any disutility. Moreover, a consumer with preference $\hat{\theta} \in (0,1)$ is indifferent between both apps, which is derived from

$$v_A - \lambda\hat{\theta} - p_A = v_B - \lambda\left(1 - \hat{\theta}\right) - p_B. \tag{2}$$

In line with the setup used by Shy and Stenbacka (2003), we assume that the value of apps is large so that $U\left(\hat{\theta}\right) \geq 0$ no matter which app is purchased. Therefore, the demands of both app providers are given by $D_A = \hat{\theta}$ and $D_B = 1 - \hat{\theta}$, respectively. Subsequently, as the value of app i can be enhanced by the add-on, we denote the value of app i as

$$v_i = \begin{cases} v, & \textit{without intelligent engine} \\ (1+\beta)v, & \textit{with intelligent engine} \end{cases} \tag{3}$$

, where v is the basic value a consumer places on the app without AI-supported features and β measures the additional benefit generated from the AI-supported add-on.

3.1 App providers, digital platform, and AI provider

In our model, as a consumer purchases apps on the digital platform, app providers pay a commission fee to the digital platform for every transaction, which depends on the commission rate and app prices. The commission rate $\hat{r}$ is exogenously given, which could range from 10%~30%[1] in most digital platforms. Therefore, letting $r \equiv 1 - \hat{r}$, the digital platform and app provider i during each transaction receive $(1 - r) \cdot p_i$ and $r \cdot p_i$, respectively. However, the AI-supported add-on has the potential to generate false information, leading to unforeseen disasters for the app market operated by the digital platform. Therefore, the digital platform imposes age restrictions on apps with AI-supported functionality for the reason of safety. Moreover,

Let $\alpha \in [0,1)$ be the ratio of users cannot use the app with suspicious risk due to AI technology and δ_i the app provider's service cost. The profit of app provider i is now given by

$$\pi_i = \begin{cases} (r \cdot p_i - \delta_i)D_i, & \textit{without intelligent engine} \\ (r \cdot p_i - \delta_i)(1-\alpha)D_i, & \textit{with intelligent engine} \end{cases} \tag{4}$$

In other words, app providers need to tradeoff the advantage and disadvantage when making the adoption decision of the AI-supported add-on. The automated content creation technology can increase consumer's willingness-to-pay for the app, but the size of user base declines due to the age limitation regulated by the digital platform.

Next, the app provider's service cost depends on its technology level. We assume that the functionality of AI-supported add-on can largely lower the app provider's service cost because its automation capability can be a game changer. For instance, audio book publishers typically need to employ voice actors to provide their services, whereas AI voice generators can perform just as effectively as voice actors, eliminating the need for hiring them. However, the AI-supported add-on is not free. The AI provider can charge app providers a royalty fee w per consumer as the cost of using the automation service provided by the intelligent engine. Therefore, regarding app provider i's service cost, we let $\delta_i = w$ and $\delta_i = c$ when the AI-supported add-on is adopted or not, respectively. Because each app provider has the option to decide whether or not to adopt the AI-supported add-on, the profit of the AI provider is

$$\pi_C = \begin{cases} 0 & \textit{, no one adopts} \\ w(1-\alpha)D_i & \textit{, only app provider i adopts} \\ w(1-\alpha)\sum D_i & \textit{, both app providers adopt} \end{cases} \tag{5}$$

[1] Commission rates for leading app stores worldwide as of July 2022

https://www.statista.com/statistics/975776/revenue-split-leading-digital-content-store-worldwide/

3.2 The timing of the game

The timing of the game is shown in Figure 1, in which the AI provider first decides on the royalty fee w in Stage I. Then, app providers make their own adoption decisions in Stage II. Without loss of generality, we assume that the AI provider initially contacts app provider A and subsequently reaches out to app provider B. In particular, we consider two cases when the AI provider contacts app providers. In Case I, as the tree structure in Figure 1 demonstrates, app provider B cannot observe the adoption decision made by app provider A. In Case II, the adoption decision made by app provider A can be observed by app provider B so that in Figure 1 the dotted line connecting the nodes in which app provider B makes adoption decision is removed. In the later subsections, the reason why we consider the two cases is explained. Finally, in Stage III, both app providers decide on their app prices simultaneously, and then consumers make their own purchasing decisions.

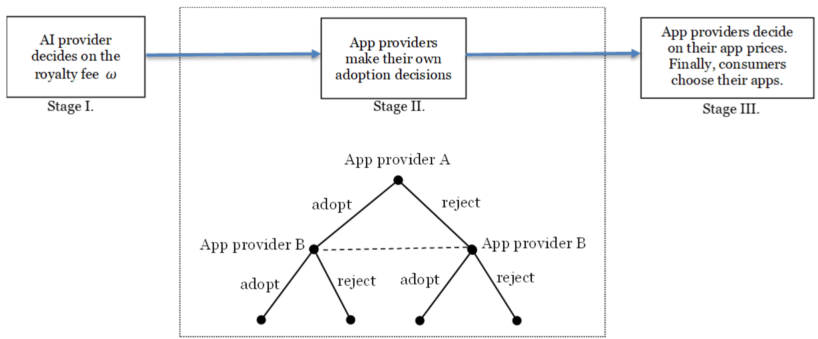


Figure 1. Game stages and sequential licensing

3.3 The adoption of AI-supported add-on

We use backward induction to solve out the adoption decision made by app providers. In Stage III, the equilibrium prices, demands, and profits under different adoption scenarios appears in the Appendix A. Subsequently, in Stage II, both app providers need to evaluate whether to adopt the AI-supported add-on. For readability, we let

$$\Delta \equiv r\beta v - w + c \tag{6}$$

for expressing equilibrium results concisely. In addition, the options 'O' and 'N' refer to the scripts with and without the AI-supported add-on, respectively. In Case I, because app provider B cannot observe app provider A's adoption decision, its equilibrium result, in fact, is the same as the case where both app providers make their adoption decisions simultaneously. Therefore, we compare the equilibrium profits in Table A4 to yield the following results for the game composed of Stage II and III.

Lemma 1. (Case I: The first mover's adoption decision cannot be observed)

(1) In case the royalty fee w is large, both app providers show no interest of adopting the AI-supported add-on. Formally, <O,O> is pure Nash equilibrium when $\Delta < 3r\lambda\left(1-\sqrt{1-\alpha}\right)$.

(2) In case the royalty fee w is small, both app providers adopt the AI-supported add-on. Formally, <N,N> is pure Nash equilibrium when $3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right) < \Delta$.

(3) In case the royalty fee w is moderate, both app providers either adopt or reject the AI-supported add-on together. In addition, their adoption decisions can be a probability distribution. Formally, <O,O> and <N,N> are pure Nash equilibria when $3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right) < \Delta < 3r\lambda\left(1-\sqrt{1-\alpha}\right)$. In addition, under mixed-strategy Nash equilibrium each app provider adopts the AI-supported add-on with the probability $1-\sigma$, where $\sigma = \frac{6r\lambda\Delta-\Delta^2-9r^2\lambda^2\alpha}{\Delta(6r\lambda\alpha-2\Delta+\Delta\alpha)}$.

According to the value of Δ, the equilibrium in Case I is shown in Figure 2. First, whether to adopt the AI-supported add-on depends on the royalty fee w. Both app providers show the interest of adopting this technology when its royalty fee is low, while rejecting this add-on is the common choice when it is costly. However, when the cost of using the intelligent engine is moderate, we show the existence of multiple equilibria. Since adopting the AI-supported add-on can incur age limitation imposed by the digital platform, the adoption decision can be investigated in terms of the degree of age limitation. Therefore, we use Figure 3 to demonstrate the equilibrium profit of an app provider under different levels of age limitation[2]. Under the mixed-strategy Nash equilibrium, each app provider has to decide on the probability of adopting the AI-supported add-on so that their rivals are indifferent when determining the probability distribution over the adoption decision. Obviously, the expected profit of each app provider under the mixed-strategy Nash equilibrium decreases when age limitation becomes stricter.

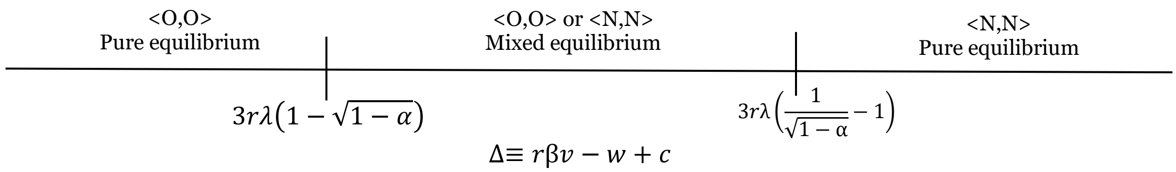


Figure 2. The equilibrium in Case I based on Δ

Subsequently, we consider Case II where the adoption decision made by app provide A cannot be observed by app provider B.

Lemma 2. (Case II: The first mover's adoption decision can be observed)

Consider the game composed of Stage II and III with complete and perfect information, both APP providers either install the AI-supported add-on when $3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right) \leq \Delta$ or reject this deal when $\Delta < 3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right)$.

The equilibrium in Case II is also demonstrated in Figure 4. Overall, the implication behind Lemma 2 is almost the same as that of Lemma 1 but contributes the tractability of our model. In Case I, the existence of multiple equilibria hinders us from finding the AI provider's decision regarding the royalty fee. Therefore, we consider Case II to further examine how AI provider decides on the royalty fee in Stage I and evaluate how the royalty fee is affected by the age limitation imposed by the digital platform.

Proposition 1. (The impact of age limitation on the AI provider)

1. With complete and perfect information, the optimal royalty fee charged by the AI provider is $w^* = r\beta v + c - 3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right)$.

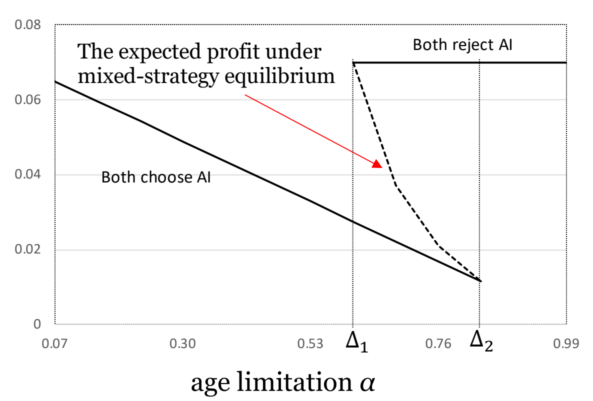
2. The optimal royalty fee decreases with the commission rate $\hat{r}$ when age limitation is loose, but the opposite holds when age limitation is strict. Formally, $\frac{\partial w^*}{\partial r} > 0$ if $\alpha < 1-\left(\frac{3\lambda}{3\lambda+\beta v}\right)^2$ but $\frac{\partial w^*}{\partial r} \leq 0$ if the opposite holds, where $r = 1-\hat{r}$.

3. Both app providers adopting the AI-supported add-on lower their product prices when age limitation becomes strict. Formally, $\frac{\partial p_i^*}{\partial \alpha} > 0$ where $i = A,B$.

When age limitation becomes stricter, the AI provider will reduce the royalty fee to relieve the cost burden for both app providers to ensure that they have incentive to adopt this add-on. Similarly, when age limitation is slight, high commission rate reduces the profits of app providers. Therefore, the AI provider also lowers the royalty fee in case app providers gain less due to high commission rate. On the other hand, when age limitation is strict, we find an interesting finding that high commission rate will drive up the royalty fee. In general, the AI provider should help these app

[2] The other two bold black lines represent the profit of each app provider when both accepting or rejecting the add-on.

providers relieve their costs by cutting the royalty fee. However, strict age limitation significantly dilutes the benefit of adopting the add-on for both app providers. Therefore, the AI provider has to charge an extremely low royalty fee to compensate the loss of both app providers due to strict age limitation. Once the commission rate increases, the AI provider can raise the royalty fee because the benefit of rejecting the add-on wanes.



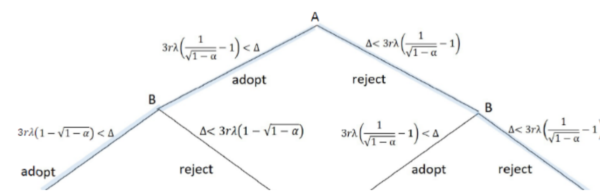


Figure 3. The expected profit of app provider i under Case I where $\Delta_1 \equiv 3r\lambda(1-\sqrt{1-\alpha})$ and $\Delta_2 \equiv 3r\lambda\left(\frac{1}{\sqrt{1-\alpha}}-1\right)$

Figure 4. The adoption decision made by app providers in Case II

3.3 Content moderation

Age limitation lowers the incentive of app providers for using the AI-supported add-on so as to lower the revenue of the AI provider. Therefore, for waiving age limitation, the AI provider can employ the technique of content moderation to ensure the auto-generated content risk-free. Let the cost of content moderation be F. Then, the revenue of the AI provider can be rewritten as

$$\hat{\pi}_C \equiv \pi_C - F, \text{ where } \alpha = 0 \tag{7}$$

Vertical Integration

Subsequently, we extend our model to examine the royalty fee when one of the app providers is merged with the AI provider. Without loss of generality, we consider that the AI provider and app provider A are merged. For convenience, the integrated firm is still known as app provider A. Therefore, the profit of the integrated firm is

$$\pi_A = r \cdot p_A(1-\alpha)D_A + I \cdot w(1-\alpha)D_B \tag{8}$$

, where the variable I is a binary indicator that expresses whether the intergraded firm licenses the AI-supported add-on to the app provider B. On the other hand, the profit of the app provider B is still the same as that in (4). Regarding the timing of the game, app provider A first decides whether to license the AI-supported add-on to app provider B. If licensing to app provider B, the integrated firm first announces the royalty fee w. Then the app provider B accepts or rejects this offer. Finally, both app providers price their apps. We first solve the case with licensing (that is, $v_A = v_B = 1 + \beta$) and then compare this result with the case without licensing to figure out the condition where the integrated firm licenses it AI-supported add-on to its competitor.

4.1 Competitive price

To being with, suppose that app provider A licenses its AI-supported add-on to app provider B, then the app prices charged by both app providers under backward induction will be

$$p_i^* = \lambda + \frac{w}{r}, \text{ where } i \in \{A, B\}. \tag{9}$$

In this case, both app providers have the same market share. Moreover, the profits of each app provider are

$$\pi_A(w) = \frac{(1-\alpha)(r\lambda+2w)}{2} \text{ and } \pi_B = \frac{r\lambda(1-\alpha)}{2}. \tag{10}$$

(9) and (10) show that the profit of app provider A increases with the royalty fee because both app providers will raise their prices. However, the indifference consumer with the utility $U(\hat{\theta})$ cannot be negative; otherwise, some of the consumers will buy nothing because the prices of both apps are too high. Consequently, $U(\hat{\theta}) = 0$ can imply the royalty fee

$$w^* = r\left(v_A - \frac{3\lambda}{2}\right). \tag{11}$$

In other words, app provider A has no incentive to announce the royalty fee less than $r\left(v_A - \frac{3\lambda}{2}\right)$ because reducing the royalty fee does not lead to increased profits for either app provider. Moreover, (9) is valid when $w \le r\left(v_A - \frac{3\lambda}{2}\right)$, which is known as competitive price in this research.

4.2 Accommodation price

When the royalty fee is equal to $r\left(v_A - \frac{3\lambda}{2}\right)$, the price of the app provider i is

$$p_i^* = (1+\beta)v - \frac{\lambda}{2}, \text{ where } i \in \{A, B\}. \tag{12}$$

In case the app provider A charges a royalty fee more than $r\left(v_A - \frac{3\lambda}{2}\right)$, the app prices given (12) remain unchanged until the royalty fee is too high. When the royalty fee w is higher than a certain threshold, the app provider B will find that increasing the app price so as to decrease the app demand is much more profitable. In case the app provider B finds that a lower demand is much better, it is, in fact, from a monopoly's point of view because the utility of the indifferent consumer is zero. Therefore, letting a consumer with preference $\overline{\theta} \in (0,1)$ is indifferent between buying app B and buying nothing, we can find out the threshold of the royalty fee by considering the maximization program of a monopoly as follows:

$$\underset{p_B}{Max}\ \pi_B = (r \cdot p_B - w)(1-\alpha)(1-\overline{\theta}) \tag{13}$$

, where $D_B = 1 - \overline{\theta} = \frac{v_B - p_B}{\lambda}$ because $U(\overline{\theta}) = v_B - \lambda(1-\overline{\theta}) - p_B = 0$. As a result, based on the optimal *monopoly* price $p_B^* = \frac{rv_B + w}{2r}$, the threshold of the royalty fee for meeting $D_B = \frac{1}{2}$ is given by

$$w^* = r(v_B - \lambda). \tag{14}$$

Thus, (12) is valid when $r\left(v(1+\beta) - \frac{3\lambda}{2}\right) < w \le r(v(1+\beta) - \lambda)$, which is known as accommodation price in this research. As long as the royalty fee is at this range, both app providers still maintain the same market share and price.

4.3 de facto Monopoly price

Finally, we consider the case where $w > r(v(1+\beta) - \lambda)$. The maximal profit the app provider B can gain is to charge

$$p_B^* = \frac{rv_B + w}{2r} \tag{15}$$

, which is derived from (13). Because a consumer with preference $\overline{\theta}$ is indifferent between buying app B and buying nothing, in this case the app provider A can extract the other consumers' surplus by charging

$$p_A = v_A - (\lambda - v_B + p_B), \tag{16}$$

which is derived from $v_A - \lambda\overline{\theta} - p_A = 0$ and $v_B - \lambda(1-\overline{\theta}) - p_B = 0$. Suppose that the app provider B always accepts the licensing contract. Then, the app provider's decision regarding the royalty fee can be formulated as

$$\underset{w}{Max}\ \pi_A = r \cdot p_A(1-\alpha)D_A + w(1-\alpha)(1-\overline{\theta}) \tag{17}$$

, where the prices are given in (15) and (16). Thus, the optimal royalty fee is given by

$$w^* = r\left(v(1+\beta) - \frac{2}{3}\lambda\right) \tag{18}$$

Though app provider A can reach its highest profit by charging this royalty fee, the app provider B's profit decreases with the royalty fee when $w \geq r\left(v(1+\beta) - \frac{3\lambda}{2}\right)$, as shown in Figure 5. Therefore, the app provider A needs to set an appropriate royalty fee such that app provider B agrees on the licensing contract. In case $r(v(1+\beta)-\lambda) < w \leq r\left(v(1+\beta) - \frac{2\lambda}{3}\right)$, (15) and (16) are valid. Moreover, we address this segment as "*de facto monopoly price*" because app provider B's price is derived from a monopoly profit maximization.

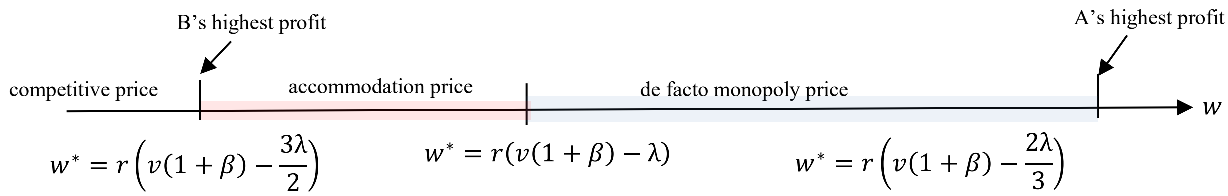


Figure 5. App provider B's pricing strategy when the royalty fee w changes

In the following, we based on Figure 5 to induce the impact of age limitation on the optimal royalty fee and each app provider's pricing strategies.

Proposition 2. In a horizontal differentiated market, if a vertical integrated firm is willing to license its AI technology to a rival app provider, the optimal royalty fee and equilibrium app prices are given by

$$w=\begin{cases} r\left(v(1+\beta)-\frac{2}{3}\lambda\right) & \text{, if } \alpha\le 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{3rv(1+\beta)(1-\alpha)-(3r\lambda-r\beta v-c)\sqrt{2(1-\alpha)}}{3(1-\alpha)} & \text{, if } 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{r^2\lambda v\cdot\left(30\beta+18-18(\beta\alpha+\alpha)\right)+12\,r\lambda c-(27-9\alpha)r^2\lambda^2-2r\beta v(2c+r\beta v)-2c^2}{18r\lambda(1-\alpha)} & \text{, if } 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \text{app provider B rejects this deal} & \text{, if } 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2<\alpha \end{cases}$$

$$,\ p_A=\begin{cases} (1+\beta)v-\frac{2}{3}\lambda & \text{, if } \alpha\le 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{3r(1+\beta)v-w}{2r}-\lambda & \text{, if } 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ (1+\beta)v-\frac{\lambda}{2} & \text{, if } 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \text{app provider B rejects this deal} & \text{, if } 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2<\alpha \end{cases}\text{, and}$$

$$p_B=\begin{cases} (1+\beta)v-\frac{1}{3}\lambda & \text{, if } \alpha\le 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{r(1+\beta)v+w}{2r} & \text{, if } 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ (1+\beta)v-\frac{\lambda}{2} & \text{, if } 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \text{app provider B rejects this deal} & \text{, if } 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2<\alpha \end{cases}$$

. Moreover, the profits of app providers are

$$\pi_A=\begin{cases} (1-\alpha)r\left((1+\beta)v-\frac{2}{3}\lambda\right) & \text{, if } \alpha\le 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{(1-\alpha)\left((8\lambda r^2+6rw)(1+\beta)v-3\left((1+\beta)rv\right)^2-4r^2\lambda^2-4\lambda rw-3w^2\right)}{4r\lambda} & \text{, if } 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \left(r\left((1+\beta)v-\frac{\lambda}{2}\right)+w\right)\left(\frac{1-\alpha}{2}\right) & \text{, if } 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \text{app provider B rejects this deal} & \text{, if } 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2<\alpha \end{cases}\text{ and}$$

$$\pi_B=\begin{cases} \frac{r\lambda(1-\alpha)}{9} & \text{, if } \alpha\le 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \frac{\left((1+\beta)rv-w\right)^2(1-\alpha)}{4r\lambda} & \text{, if } 1-\frac{9}{2}\cdot\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \left(r\left((1+\beta)v-\frac{\lambda}{2}\right)-w\right)\left(\frac{1-\alpha}{2}\right) & \text{, if } 1-2\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2\le\alpha\le 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2 \\ \text{app provider B rejects this deal} & \text{, if } 1-\left(1-\frac{r\beta v+c}{3r\lambda}\right)^2<\alpha \end{cases}.$$

We find that the strictest age limitation will hinder the app provider B from accepting the license proposed by the integrated firm. In addition, the integrated firm has to adjust the royalty fee such that the app provider B is willing to accept this contract. In other words, the optimal royalty fee depends on the level of age limitation. In case age limitation is absent or extremely slight, the integrated firm can even employ the license to reach its highest profit. This case shows that an integrated firm in a horizontal differentiated market can raise its profit via licensing its own exclusive technology to other competitors. Another interesting finding is related to the degree of horizontal differentiation. In general, the higher degree of horizontal differentiation between both app providers should benefit themselves. However, our result indicates that this cannot always hold under vertical integration. The profit of the integrated firm under different royalty fees decreases when the degree of horizontal differentiation increases. With the licensing contract, the integrated firm can coordinate both providers' pricing strategies. Therefore, the increased degree of horizontal differentiation simply lower the profits of both app providers because serving the consumers who dislike both products becomes larger.

Conclusion

This study delves into the nuanced landscape of licensing AI-supported functionalities in app markets, particularly under the constraints of user base restrictions. The advent of generative AI has prompted regulatory measures aimed at protecting users, which, paradoxically, could stifle revenue streams for applications that stand to gain from the novel functionalities offered by AI. Through a comprehensive analysis, we have explored the strategic maneuvers of AI providers in navigating these challenges. Our findings illuminate the complex interplay between the need for widespread adoption of AI technologies and the imperative to manage intellectual property rights amidst evolving regulatory and market dynamics.

The examination reveals a delicate equilibrium AI providers must achieve. They need to judiciously set their licensing fees, balancing the allure of broad utilization of their technologies against the critical need to uphold control over their innovations and adapt to a landscape marked by user base restrictions. This balance is pivotal in ensuring the continued viability and profitability of AI functionalities within the app market, which is increasingly governed by both competitive and regulatory pressures.

Moreover, the study extends into the realm of vertical integration, assessing its implications for licensing strategies and market positioning. It underscores how vertical integration can serve as a strategic lever for AI providers, enabling them to navigate competitive pressures while optimizing their licensing models in the face of user base restrictions.

Our research contributes to the broader discourse on technology adoption and licensing, offering novel insights into the strategic considerations unique to the digital app market. It not only enriches our understanding of the strategic dimensions of technology licensing but also highlights the significant impact of regulatory and competitive forces on the deployment and commercialization of AI technologies. As the digital landscape evolves, so too will the strategies employed by AI providers and app developers, underscoring the importance of continued research in this dynamic field.

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Appendix A. Equilibrium prices and profit levels

Based on (2), $\hat{\theta} = \frac{1}{2} + \frac{v_A - v_B - p_A + p_B}{2\lambda}$. Subsequently, the demand of each app provider, $D_A = \hat{\theta}$ and $D_B = 1 - \hat{\theta}$, can be obtained by employing (3), as shown in Table A1. After the adoption decision is made, each provider decides on their prices simultaneously. In other words, based on (4), we solve $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$ simultaneously where D_A and D_B are given in Table A1. The equilibrium prices are given in Table A2, and Table A3 shows the demands of app providers after incorporating the equilibrium prices in Table A2 into the demand functions in Table A1. Finally, the profit levels under the equilibrium prices are given in Table A4.

Table A1. Demands of app providers under different adoption scenarios

Provider A	Provider B			
	O (Without AI add-on)		N (With AI add-on)	
O	$\frac{1}{2} + \frac{p_B - p_A}{2\lambda}$	$\frac{1}{2} - \frac{p_B - p_A}{2\lambda}$	$\frac{1}{2} + \frac{p_B - p_A - \beta v}{2\lambda}$	$\frac{1}{2} - \frac{p_B - p_A - \beta v}{2\lambda}$
N	$\frac{1}{2} + \frac{p_B - p_A + \beta v}{2\lambda}$	$\frac{1}{2} - \frac{p_B - p_A + \beta v}{2\lambda}$	$\frac{1}{2} + \frac{p_B - p_A}{2\lambda}$	$\frac{1}{2} - \frac{p_B - p_A}{2\lambda}$

Table A2. App prices under different adoption scenarios

Provider A	Provider B			
	O (Without AI add-on)		N (With AI add-on)	
O	$\lambda + \frac{c}{r}$	$\lambda + \frac{c}{r}$	$\lambda - \frac{r\beta v - w - 2c}{3r}$	$\lambda + \frac{r\beta v + 2w + c}{3r}$
N	$\lambda + \frac{r\beta v + 2w + c}{3r}$	$\lambda - \frac{r\beta v - w - 2c}{3r}$	$\lambda + \frac{\omega}{r}$	$\lambda + \frac{\omega}{r}$

Table A3. Equilibrium market shares under different adoption scenarios

Provider A	Provider B			
	O (Without AI add-on)		N (With AI add-on)	
O	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{\Delta}{6r\lambda}$	$\frac{1}{2} + \frac{\Delta}{6r\lambda}$
N	$\frac{1}{2} + \frac{\Delta}{6r\lambda}$	$\frac{1}{2} - \frac{\Delta}{6r\lambda}$	$\frac{1}{2}$	$\frac{1}{2}$

$\Delta \equiv r\beta v - w + c$

Table A4. Equilibrium profit levels under different adoption scenarios

Provider A	Provider B			
	O (Without AI add-on)		N (With AI add-on)	
O	$\frac{r\lambda}{2}$	$\frac{r\lambda}{2}$	$\frac{(3r\lambda - \Delta)^2}{18r\lambda}$	$\frac{(3r\lambda + \Delta)^2(1-\alpha)}{18r\lambda}$
N	$\frac{(3r\lambda + \Delta)^2(1-\alpha)}{18r\lambda}$	$\frac{(3r\lambda - \Delta)^2}{18r\lambda}$	$\frac{r\lambda(1-\alpha)}{2}$	$\frac{r\lambda(1-\alpha)}{2}$

Due to space limitations, if you are interested in other proofs, please contact us directly.