On Effects of Continuous Event Approximations on Sensitivities and Optimization of Multibody Systems with Friction

Adwait Verulkar, Adrian Sandu, Corina Sandu and Daniel Dopico

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On Effects of Continuous Event Approximations on Sensitivities and Optimization of Multibody Systems with Friction

Adwait Verulkar[†], **Adrian Sandu**[#], **Corina Sandu**[†], **Daniel Dopico**[*]

[†] Department of Mechanical Engineering,
Virginia Tech, Blacksburg, VA 24060
[adwaitverulkar, csandu]@vt.edu

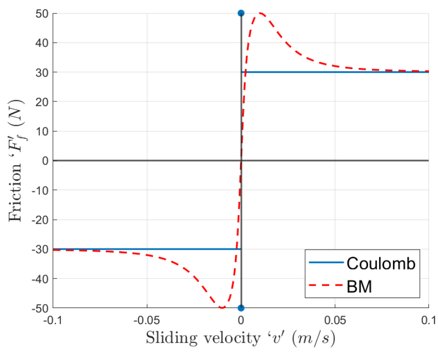
[#] Department of Computer Science,
Virginia Tech, Blacksburg, VA 24060
asandu7@vt.edu

[*]Department of Naval and Industrial Engineering,
University of A Coruña, A Coruña, Spain, 15071
ddopico@udc.es

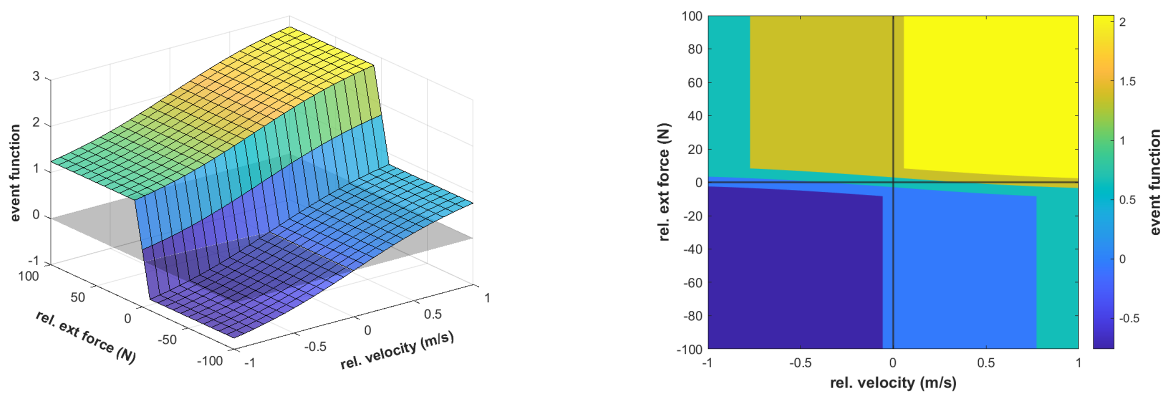
Abstract

This paper dives into the domains of sensitivity analysis and optimization of multibody systems with frictional interactions. It explores the nuances associated with the approximation of discrete events in friction model using high derivative continuous functions. Such approximations may work well when it comes to computing dynamics, however the sensitivities may be very different. The study presents a comparative analysis between a discrete event-based Coulomb friction model and a continuous approximation Brown-McPhee friction model [1].

$$
\begin{aligned}
F_{\text{BM}} &= F_n \left[\mu_d \tanh\left(\frac{4v_{\text{rel}}}{v_t}\right) + \frac{(\mu_s - \mu_d)\left(\frac{v_{\text{rel}}}{v_t}\right)}{\left[\left(\frac{v_{\text{rel}}}{2v_t}\right)^2 + \frac{3}{4}\right]^2} \right] \\
F_{\text{dyn}} &= \mu_d F_n \\
F_{\text{sat}} &= \mu_s F_n \\
F_{\text{Coulomb}} &= \begin{cases} F_{\text{ext}} & \text{if} \quad |v_{\text{rel}}| \le \varepsilon \quad \text{and} \quad F_{\text{ext}} \le F_{\text{sat}} \\ F_{\text{dyn}} & \text{if} \quad |v_{\text{rel}}| \le \varepsilon \quad \text{and} \quad F_{\text{ext}} > F_{\text{sat}} \\ F_{\text{dyn}} & \text{if} \quad |v_{\text{rel}}| > \varepsilon \end{cases}
\end{aligned}
\tag{1}
$$



Equation (1) compares the non-viscous Brown-McPhee friction model with respect to the event-based Coulomb friction model. Coulomb friction exhibits discontinuous events during stiction to friction transition which are approximated through a steep continuous function by the Brown McPhee model. A validation study must be conducted by comparing the sensitivities obtained by differentiating the Brown McPhee model and those obtained through event-based Coulomb model. To compute the sensitivities with respect to any hybrid dynamic model, an event function is required. Additionally, the event function must have non-zero slope during the event trigger (zero crossing) [2]. A function was designed that is negative when both the relative external force ($|F_{\text{ext}}| - F_{\text{sat}}$) and relative sliding velocity ($|v_{\text{rel}}| - \varepsilon$) are negative. Figure 1 shows the event function used for the study.

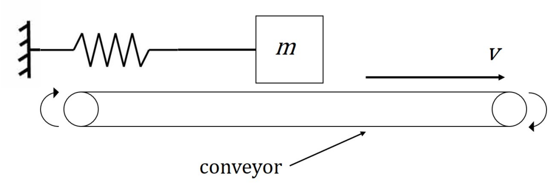


(a) Event function surface

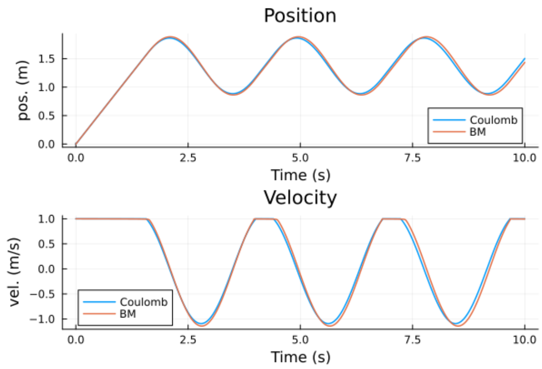
(b) Event function negative only in quadrant III

Figure 1: Event function $1 + \tanh(x) + \tanh(y)$ for stiction to dynamic friction transition

For numerical results, the Rabinowicz experiment serves as a good benchmark study. It consists of a block on a rough conveyor with a spring attached to it. The movement of the conveyor creates a cyclic stiction to friction transition that helps compare various friction models.



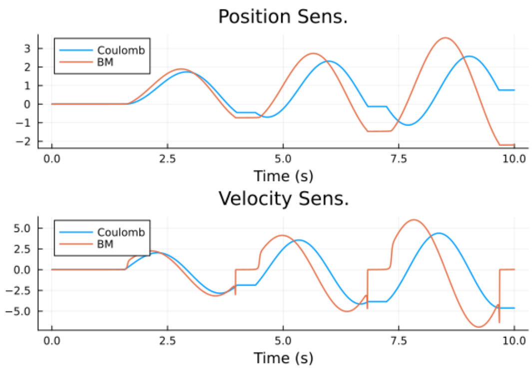
(a) Rabinowicz experiment schematic



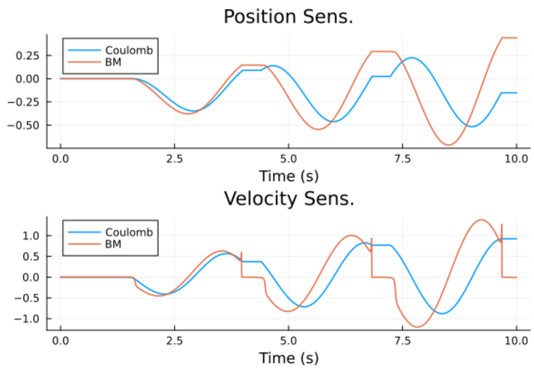
(b) Block position and velocity plots

Figure 2: Dynamics of Rabinowicz experiment

Figure 2 shows the dynamics of this system and plots the block position and velocity as a function of time. The plot compares the dynamics when the friction is modeled using Coulomb and Brown McPhee models. The dynamic plots are very similar for the same experiment parameters: $m = 1$ kg, $k = 10$ N/m, $v = 1.0$ m/s, $\mu_s = 0.8$, and $\mu_d = 0.7$.



(a) Sensitivities with respect to block mass



(b) Sensitivities with respect to spring constant

Figure 3: Comparison of sensitivities with respect to static parameters

However, the same cannot be said about model sensitivities. Figure 3 plots the dynamic sensitivities of the model with respect to 2 static parameters viz. block's mass m and spring constant k. It can be seen that the velocity sensitivities computed through Brown McPhee model are zero during the stiction phase thereby resulting in constant position sensitivities. However, the event-based Coulomb friction model exhibits position and velocity sensitivities that are a non-zero constant during stiction. The magnitudes of the sensitivities computed through both models are comparable which may lead to similar gradients with respect to static parameters. However, the differences in sensitivities may play a significant role in dynamic optimization case studies such as optimal control or dynamic estimation problems.

References

[1] Peter Brown and John McPhee. A Continuous Velocity-Based Friction Model for Dynamics and Control with Physically Meaningful Parameters. *Journal of Computational and Nonlinear Dynamics*, 11(5):1–6, 2016.

[2] Sebastien Corner, Corina Sandu, and Adrian Sandu. Modeling and sensitivity analysis methodology for hybrid dynamical system. *Nonlinear Analysis: Hybrid Systems*, 31:19–40, 2019.