Problem Reduction as a general epistemic reasoning method

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We introduce a general (epistemic) reasoning method based on problem reduction and show its use and discuss its justifiability in several disciplines.

To introduce our concept we rephrase and extend the question-answer approach of A. Blass [B]. We consider a problem (problem domain) P = (I, S, A) consisting of set of problem instances I, set of potential solutions S and acceptability relation A, where A(i,s) means that a particular solution s in S is acceptable for a problem instance i in I.

Problem reduction occurs e.g. when a problem solution is delegated to an expert, or a client computation asks a server to do a part of a job, or an agent with limited resources asks agent with richer knowledge, or a real world problem is reduced to a (computational) model.

Assume we have two problem domains $\boldsymbol{P_1}=(I_1,S_1,A_1)$ (sometimes called target domain) and $\boldsymbol{P_2} = (I_2,S_2,A_2)$ (sometimes called source domain). We consider a typical scenario: assume we are not able (or it is very complex or very expensive) to solve problem instances from I_1. Moreover, assume there is a problem domain $\boldsymbol{P_2}$ where we have methods to find acceptable solutions (efficient, cheaper). If we happen to efficiently reduce problem instances from I_1 to problem instances of I_2 in such a way that acceptable solutions in S_2 can be transformed to acceptable solutions to original problem instance, we are done. There is a wide space for what acceptability can mean. It can be e.g. correct, reasonable, reliable, etc.

Problem reduction (PR). Reduction of a problem $\boldsymbol{P_1}$ to a problem $\boldsymbol{P_2}$ consists of a pair of mappings *(r,t): r* maps problem instances i_1 in I_1 of $\boldsymbol{P_1}$ to problem instances r(i_1) in I_2 of $\boldsymbol{P_2}$ and t maps solutions s_2 in S_2 to solutions t(s_2) in S_1 in such a way, that an acceptable (in the sense of relation A_2) solution s_2 to instance r(i_1) is transferred to an solution t(s_2) which is A_1-acceptable to original problem instance i_1. Formally we require: for all i_1 in I_1 and s_2 in S_2

$$A_2(\mathrm{r}(i_1), s_2) \text{ implies } A_1(i_1, \mathrm{t}(s_2)) \text{ holds true.} \qquad \text{(PRi)}$$

Motivated by [Hr] we combine decision and search problems, and assume every set of solutions contains an extra element “nas” = “no_acceptable_solution” and we require the above implication should be valid also for s_2 = nas$_2$ and t(nas$_2$) = nas$_1$. This helps us to avoid empty fulfillment of (PRi) implication and to preserve category theoretical character of problem reductions.

Our approach generalizes analogical reasoning [A], in that we show that along of similarity it works also under some quite complementary situations.

Comenius [C] was the first western theorist to formulate an educational theory “according to nature”. Comenius formulated the principle that „the exact order of instruction must be borrowed from nature“. We show that this can be understood as a use of our problem reduction reasoning method and justified through centuries.

Following [He] we can read following question-answer:
Heidegger: ... The role of philosophy in the past has been taken ever today by the sciences ...
… Philosophy [today] dissolves into individual sciences: psychology, logic, political science.
SPIEGEL: And what now takes the place of Philosophy?
Heidegger: Cybernetics.
We will show that our general (epistemic) reasoning method based on problem reduction can be used to understanding cybernetics as modeling of dynamic processes with feedback. This can shed light to Heidegger's answer.

Another application comes from modeling and abstraction in System Sciences. Inspired by Peter Senge [S], we propose the correlation of the organizational analysis’ depth with the different type of possible actions (solutions). First reducing problem instances from events analysis to patterns of behavior analysis we reach finally systemic structure analysis (on problem instances side). Finding an acceptable generative action and transforming back along the solution side to responsive action and finally to reactive action closes the use of our back and forth reduction and translation.

We consider also use in the management by objectives model in organizational envisioning and narration. These were justified empirically in a certain degree of acceptance of solution. Our reasoning works also under uncertainty.

Problem reduction itself, as a reasoning method, can be quite challenging (similarly to finding mathematical proofs). Nevertheless we believe that advantages of finding P_2, r, t and proving implication (PRi) for solving P_1 are worth of these difficulties.

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