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August 12, 2022

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Abstract

This paper provides a generalized definition of the unification of Order-Sorted Feature (OSF) terms that considers a *fuzzy subsumption relation* between sort symbols rather than an ordinary (crisp) one. In this setting the unifier of two OSF terms is associated with a subsumption degree. We refer to the problem of unifying two OSF terms and computing the associated subsumption degree as *fuzzy OSF term unification*.

1 Introduction

Approximate reasoning grounded on fuzzy relations is a research area that has been investigated extensively. Early work includes Ying’s logic for approximate reasoning [22] and the first papers on similarity-based logic programming [6, 12, 21]. One motivation behind the similarity-based approaches was to model a form of reasoning that may be referred to as *reasoning by analogy or similarity*. For example, this may be achieved by relaxing the *equality* constraint on two functor symbols, when unifying two first-order terms (FOTs), to the flexible constraint that they must be *similar*. This kind of relaxed unification is generally referred to as *weak unification*.

This research line has been extended in a number of ways, such as via proximity-based approaches [14, 16, 17]; moreover, weak unification has been implemented in fuzzy logic programming systems such as Bousi~Prolog [15] and FASILL [13]. Ait-Kaci and Pasi [3] have presented a procedure for weak unification that not only tolerates different (but similar) functor symbols, but also allows the unification of FOTs with a different number and possibly a different order of arguments. This work has been generalized to proximity relations [20], and a possible incorporation in Bousi~Prolog has been proposed [10].

The work by Ait-Kaci and Pasi was preliminary towards the definition of similarity-based reasoning with Order-Sorted Feature (OSF) logic, a knowledge representation language developed by Ait-Kaci [4], which has found applications in constraint logic programming and computational linguistics (see, e.g., [7] for feature-based logical formalisms and their use in linguistics). One advantage of OSF logic is that its unification algorithm takes into account a *subsumption* (IS-A) ordering between sorts, which enables a single unification step to potentially replace several resolution steps, possibly leading to more efficient computations [2, 9].

This paper provides a generalized definition of subsumption between OSF terms and of the unification of OSF terms that considers a *fuzzy subsumption relation between sorts symbols* rather than an ordinary (crisp) one. We refer to the problem of unifying two OSF terms with an underlying fuzzy subsumption relation and computing the associated subsumption degree as *fuzzy OSF term unification*. One benefit of the present approach is that the unification itself can be performed with the same rules of (crisp) OSF term unification. The introduction of fuzziness in the sort signature is meant to provide more modeling flexibility by allowing to represent imprecise knowledge. Ultimately, the goal of this research is to define a framework based on OSF logic that allows to deal with imperfect knowledge and information, and that enables approximate matching in applications such as information retrieval and question answering.

2 OSF Terms and Fuzzy Sort Subsumption

OSF logic [4] is based on *sort symbols* and *feature symbols*: sort symbols denote concepts or classes of objects, while feature symbols are interpreted as properties or attributes of sorts. These symbols together with *variables* (or *coreference tags*) are used to construct OSF terms.

Definition 2.1 (OSF Terms, $Root(\psi)$ and $Tags(\psi)$). Let \mathcal{V} be a set of variables, \mathcal{S} be a set of sort symbols and \mathcal{F} be a set of feature symbols. Then (i) a variable $X \in \mathcal{V}$ is an OSF term; (ii) if $X \in \mathcal{V}$ and $s \in \mathcal{S}$, a sorted variable $X : s$ is an OSF term; (iii) if ψ_1, \dots, ψ_n are OSF terms, $f_1, \dots, f_n \in \mathcal{F}$, $X \in \mathcal{V}$ and $s \in \mathcal{S}$, then $X : s(f_1 \rightarrow \psi_1, \dots, f_n \rightarrow \psi_n)$ is an OSF term. The variable X is called the root of the term ψ and denoted $Root(\psi)$. The set of variables appearing in the term ψ is denoted $Tags(\psi)$.

Example 2.2 (OSF Term). An example of an OSF term is the following:

$$\psi = X_0 : movie \left(\begin{array}{l} directed_by \rightarrow X : director (name \rightarrow X_1 : string), \\ written_by \rightarrow X \end{array} \right) \quad (\psi)$$

where *movie*, *director* and *string* are sort symbols, *name*, *written_by* and *directed_by* are feature symbols, and X, X_0 (the root) and X_1 are variables denoting objects of the domain (e.g., X must denote a director). Informally, this term represents movies that are written and directed by the same individual. OSF terms generalize FOTs: for instance, the FOT `movie("Psycho", "Hitchcock")` can be translated into OSF syntax as `movie(1 → "Psycho", 2 → "Hitchcock")`, or extended using feature symbols into `movie(name → "Psycho", directed_by → "Hitchcock")` for increased interpretability (variables are omitted for readability).

In the regular setting, the set \mathcal{S} of sort symbols is ordered by an IS-A *subsumption relation* \preceq , formally a finite lattice. In this paper we generalize this relation to a *fuzzy subsumption relation*, that is, we consider a fuzzy lattice (\mathcal{S}, \preceq) on the (finite) set \mathcal{S} of sort symbols, meaning that \preceq is a fuzzy binary relation (a function) $\preceq : \mathcal{S} \times \mathcal{S} \rightarrow [0, 1]$ that satisfies:

$$\forall s \in \mathcal{S}, \preceq(s, s) = 1 \quad (\text{Fuzzy Reflexivity})$$

$$\forall s, s' \in \mathcal{S}, \text{ if } s \preceq s' \text{ and } s' \preceq s, \text{ then } s = s' \quad (\text{Fuzzy Antisymmetry})$$

$$\forall s_0, s_1, s_2 \in \mathcal{S}, \preceq(s_0, s_2) \geq \min(\preceq(s_0, s_1), \preceq(s_1, s_2)) \quad (\text{Max-Min Transitivity})$$

and such that the *fuzzy greatest lower bound* (GLB) $s_0 \wedge s_1$ exists for each $s_0, s_1 \in \mathcal{S}$, where $s_0 \wedge s_1$ is defined as the unique $s \in \mathcal{S}$ such that $s \preceq s_0$, $s \preceq s_1$, and, for all $s' \in \mathcal{S}$, if $s' \preceq s_0$ and $s' \preceq s_1$, then $s' \preceq s$ [8, 18]. We also assume that the bottom (\perp) and top (\top) elements of the lattice are such that, for any $s \in \mathcal{S}$, $\preceq(\perp, s) = \preceq(s, \top) = 1$.

The GLB $s \wedge s'$ of two sorts $s, s' \in \mathcal{S}$ is associated with a *subsumption degree* defined as $GLBDegree(s, s') = \min(\preceq(s \wedge s', s), \preceq(s \wedge s', s'))$. We write $s \wedge_\alpha s'$ to express that the (fuzzy) GLB $s \wedge s'$ is associated with the subsumption degree $GLBDegree(s, s') = \alpha$.

Example 2.3 (Fuzzy GLBs). Consider the weighted directed acyclic graph (DAG) of Figure 1. Its (max-min) transitive closure [11] is a fuzzy lattice (the bottom and top elements are omitted in the figure). For example, the (fuzzy) GLB of u and t is $u \wedge t = q$ and $GLBDegree(u, t) = 0.6$.

¹We write $s \preceq s'$ to abbreviate $\preceq(s, s') > 0$.

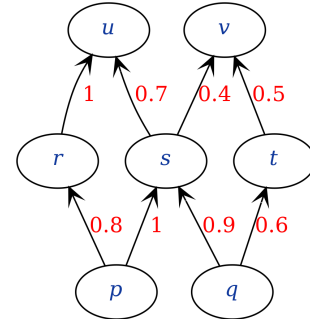


Figure 1: Fuzzy Lattice

Proposition 2.4 (GLBs in the Support of a Fuzzy Lattice). *Let $|\preceq|$ denote the support of the fuzzy binary relation \preceq , which is defined as $|\preceq| = \{(s, s') \in \mathcal{S} \times \mathcal{S} \mid \preceq(s, s') > 0\}$. It holds that (i) if (\mathcal{S}, \preceq) is a fuzzy lattice, then $(\mathcal{S}, |\preceq|)$ is a lattice, and (ii) fuzzy GLBs in (\mathcal{S}, \preceq) correspond to GLBs in $(\mathcal{S}, |\preceq|)$, i.e., $s \wedge s' = s \wedge s'$ for all $s, s' \in \mathcal{S}$, where \wedge and \wedge are the (fuzzy) GLB operations on (\mathcal{S}, \preceq) and $(\mathcal{S}, |\preceq|)$, respectively.*

Thanks to this fact it is possible to compute fuzzy GLBs exactly as we would in an ordinary lattice, which can be performed in constant time after a linear time preprocessing of the DAG representing the sort subsumption relation [1]. On the other hand, for any two sorts $s, s' \in \mathcal{S}$, the values $\preceq(s, s')$ and $GLBDegree(s, s')$ can be computed in linear time (in the size of the weighted DAG representing the fuzzy sort subsumption) with a shortest-paths-like algorithm that can be optimized thanks to the same preprocessing step [19].

3 Fuzzy OSF Term Subsumption and Unification

The definition of OSF terms given above does not rule out the presence of redundant or even contradictory information (e.g., consider the OSF term $s(f \rightarrow s_0, f \rightarrow s_0, f \rightarrow s_1)$, which is contradictory if $s_0 \wedge s_1 = \perp$). OSF terms that are well-behaved to this regard are called *normal OSF terms* and are defined as follows [4]. Unless explicitly stated otherwise, *it is always assumed that an OSF term is in normal form*².

Definition 3.1 (Normal OSF term). An OSF term $\psi = X : s(f_1 \rightarrow \psi_1, \dots, f_n \rightarrow \psi_n)$ is *normal* (or *in normal form*) if: (i) $s \neq \perp$, (ii) the features f_1, \dots, f_n are pairwise distinct, (iii) each ψ_i is in normal form, and (iv) for all $Y \in Tags(\psi)$, there is at most one occurrence of Y in ψ such that Y is the root variable of a non-trivial OSF term (i.e., different from $Y : \top$).

For an OSF term ψ in normal form and $X \in Tags(\psi)$, we let $Sort_\psi(X)$ be the most specific sort s such that $X : s$ appears in ψ . The expression $X.f \doteq_\psi Y$ means that there is a feature f pointing from a subterm of ψ with root X to a subterm of ψ with root Y .

The fuzzy subsumption relation between sorts can be extended to (normal) OSF terms according to the following definition, which generalizes the one for OSF term subsumption [4]³.

Definition 3.2 (Fuzzy OSF Term Subsumption). An OSF term ψ_1 is *subsumed by an OSF term ψ_2 with degree $\alpha \in [0, 1]$* (denoted $\psi_1 \preceq_\alpha \psi_2$) if there is a mapping $h : Tags(\psi_2) \rightarrow Tags(\psi_1)$ such that (i) $h(Root(\psi_2)) = Root(\psi_1)$, (ii) $\alpha = \min\{\preceq(Sort_{\psi_1}(h(X)), Sort_{\psi_2}(X)) \mid X \in Tags(\psi_2)\}$, and (iii) if $X.f \doteq_{\psi_2} Y$, then $h(X).f \doteq_{\psi_1} h(Y)$. The fuzzy relation \preceq on sorts is extended to OSF terms by letting $\preceq(\psi_1, \psi_2) = \alpha$ if ψ_1 is subsumed by ψ_2 with degree α .

Example 3.3 (Fuzzy OSF Term Subsumption). Consider the fuzzy lattice of Figure 1 and the terms $\psi_1 = X_0 : q(f \rightarrow X_1 : s(g \rightarrow X_0))$ and $\psi_2 = Y_0 : v(f \rightarrow Y_1 : u(g \rightarrow Y_2 : t))$. Then $\psi_1 \preceq_{0.5} \psi_2$ via the mapping $h : Tags(\psi_2) \rightarrow Tags(\psi_1)$ defined by $h(Y_0) = h(Y_2) = X_0$ and $h(Y_1) = X_1$, since h preserves the structure of ψ_2 and each tag $Y \in Tags(\psi_2)$ is mapped to a tag $h(Y) \in Tags(\psi_1)$ such that $Sort_{\psi_1}(h(Y)) \preceq Sort_{\psi_2}(Y)$; in particular $\preceq(q, v) = 0.5$.

Proposition 3.4 (Fuzzy OSF Term Subsumption). *The fuzzy subsumption relation between OSF terms is a fuzzy partial order (modulo variable renaming), i.e., it satisfies [Fuzzy Reflexivity](#), [Fuzzy Antisymmetry](#) (modulo variable renaming) and [Max-Min Transitivity](#).*

²The normal form of an OSF term can be computed by applying the OSF constraint normalization rules to its corresponding OSF clause. These notions will be defined shortly.

³To be more precise, subsumption between OSF terms is defined semantically in [4], although an equivalent characterization based on homomorphisms between OSF graphs – an alternative syntactic representation of normal OSF terms – is also given. Also see the definition of subsumption between feature structures in [7].

The unification of two OSF terms – an operation that aims to combine two OSF terms in a consistent way – is better presented by introducing an alternative syntactic representation, namely *OSF clauses* [4].

Definition 3.5 (OSF Clause). An OSF clause ϕ is a conjunctive set of expressions (*constraints*) of the form $X : s$, $X \doteq X'$, and $X.f \doteq X'$. The set of variables occurring in ϕ is denoted $Tags(\phi)$, while $\phi[X/Y]$ is the OSF clause obtained by replacing all occurrences of Y with X .

Informally, the constraint $X : s$ means that the value assigned to X is of sort s ; $X \doteq X'$ means that the same value is assigned to the variables X and X' ; while $X.f \doteq X'$ means that applying the feature f to the value assigned to X returns the value assigned to X' .

Any OSF term ψ can be translated into an equivalent OSF clause $\phi(\psi)$. For example, the following is the OSF clause corresponding to the term ψ from Example 2.2:

$$\phi(\psi) = \left\{ \begin{array}{l} X_0 : movie, \quad X_0.directed_by \doteq X, \quad X_0.written_by \doteq X, \\ X : director, \quad X.name \doteq X_1, \quad X_1 : string \end{array} \right\}.$$

The next fact follows from Propositions 2.4 and 3.4 and the analogous result for (crisp) OSF term unification [4].

Proposition 3.6 (Fuzzy OSF Term Unification). *Two OSF terms ψ_1 and ψ_2 can be unified by non-deterministically applying any applicable constraint normalization rule (Figure 2) to the clause $\phi(\psi_1) \cup \phi(\psi_2) \cup \{Root(\psi_1) \doteq Root(\psi_2)\}$ until none applies. The resulting clause can be translated back into an OSF term ψ , called the unifier of ψ_1 and ψ_2 (the term $X : \perp$ if the unification fails). The term ψ is the (fuzzy) GLB of ψ_1 and ψ_2 (up to variable renaming) with respect to the fuzzy subsumption relation of Definition 3.2 and is denoted $\psi_1 \wedge \psi_2$.*

Sort Intersection	Feature Functionality
$\frac{\phi \cup \{X : s, X : s'\}}{\phi \cup \{X : s \wedge s'\}}$	$\frac{\phi \cup \{X.f \doteq Y, X.f \doteq Y'\}}{\phi \cup \{X.f \doteq Y, Y \doteq Y'\}}$
Inconsistent Sort	Tag Elimination
$\frac{\phi \cup \{X : \perp\}}{fail}$	$\frac{\phi \cup \{X \doteq Y\}}{\phi[X/Y] \cup \{X \doteq Y\}} \quad [Y \in Tags(\phi)]$

Figure 2: OSF Constraint Normalization Rules

The (fuzzy) subsumption relation plays an essential role when unifying two OSF terms, as highlighted by the rules **Sort Intersection** and **Inconsistent Sort**. In particular, the unification fails if at any point the constraint $X : \perp$ is reached, possibly by applying the rule **Sort Intersection** to two constraints $X : s$ and $X : s'$ such that $s \wedge s' = \perp$.

We conclude by specifying how to compute the subsumption degree associated with the GLB of two OSF terms. Let ϕ be the OSF clause resulting from the application of the constraint normalization rules to the clause $\phi(\psi_1) \cup \phi(\psi_2) \cup \{Root(\psi_1) \doteq Root(\psi_2)\}$, and ψ be the OSF term corresponding to ϕ , so that $\psi = \psi_1 \wedge \psi_2$. As shown in Example 3.7, the mappings $h_1 : Tags(\psi_1) \rightarrow Tags(\psi)$ and $h_2 : Tags(\psi_2) \rightarrow Tags(\psi)$ witnessing the relations $\psi \preceq \psi_1$ ⁴ and $\psi \preceq \psi_2$ can be computed from the clause ϕ . The subsumption degree $\preceq(\psi, \psi_1)$ is given by $\min\{\preceq(Sort_\psi(h_1(X)), Sort_{\psi_1}(X)) \mid X \in Tags(\psi_1)\}$ and similarly for $\preceq(\psi, \psi_2)$. Finally, the

⁴We write $\psi_1 \preceq \psi_2$ to abbreviate $\preceq(\psi_1, \psi_2) > 0$.

subsumption degree associated with the GLB $\psi = \psi_1 \wedge \psi_2$ is equal to $GLBDegree(\psi_1, \psi_2) = \min(\preceq(\psi, \psi_1), \preceq(\psi, \psi_2))$.

Example 3.7 (Fuzzy OSF Term Unification). Consider the OSF terms

$$\psi_1 = Y_0 : u (f \rightarrow Y_1 : v (g \rightarrow Y_0, h \rightarrow Y_2 : r)) \text{ and } \psi_2 = X_0 : v (f \rightarrow X_1 : u (g \rightarrow X_2 : t))$$

and the corresponding clauses $\phi(\psi_1) = \{Y_0 : u, Y_0.f \doteq Y_1, Y_1 : v, Y_1.g \doteq Y_0, Y_1.h \doteq Y_2, Y_2 : r\}$ and $\phi(\psi_2) = \{X_0 : v, X_0.f \doteq X_1, X_1 : u, X_1.g \doteq X_2, X_2 : t\}$. An application of the rules of Figure 2 to $\phi(\psi_1) \cup \phi(\psi_2) \cup \{X_0 \doteq Y_0\}$ (with Figure 1 as the fuzzy subsumption) yields

$$\phi = \{X_0 : q, X_0.f \doteq X_1, X_1 : s, X_1.g \doteq X_0, X_1.h = Y_2, Y_2 : r, X_0 \doteq Y_0, X_0 \doteq X_2, X_1 \doteq Y_1\}$$

or an equivalent clause. The set $Tags(\phi)$ can be partitioned into the equivalence classes $[X_0]_{\doteq} = \{X_0, X_2, Y_0\}$, $[X_1]_{\doteq} = \{X_1, Y_1\}$ and $[Y_2]_{\doteq} = \{Y_2\}$. A new tag can be introduced for each class, say Z_0 for $[X_0]_{\doteq}$, Z_1 for $[X_1]_{\doteq}$ and Z_2 for $[Y_2]_{\doteq}$. The unifier of ψ_1 and ψ_2 can be constructed from ϕ using these new variables, resulting in

$$\psi = Z_0 : q (f \rightarrow Z_1 : s (g \rightarrow Z_0, h \rightarrow Z_2 : r)).$$

The functions $h_i : Tags(\psi_i) \rightarrow Tags(\psi)$ ($i = 1, 2$) can be defined by mapping each variable to the tag associated with its equivalence class. Thus $h_2(X_0) = h_2(X_2) = h_1(Y_0) = Z_0$, $h_2(X_1) = h_1(Y_1) = Z_1$ and $h_1(Y_2) = Z_2$. The subsumption degrees are $\preceq(\psi, \psi_1) = \min\{\preceq(q, u), \preceq(s, v), \preceq(r, r)\} = 0.4$ and $\preceq(\psi, \psi_2) = \min\{\preceq(q, v), \preceq(s, u), \preceq(g, t)\} = 0.5$. Overall, the subsumption degree associated with the GLB $\psi = \psi_1 \wedge \psi_2$ is $GLBDegree(\psi_1, \psi_2) = 0.4$. The unification is depicted in Figure 3, where ψ_1 corresponds to the blue graph, ψ_2 to the red graph, ψ to the yellow graph, and the dashed arrows represent the mappings.

4 Future Work

The generalized definition of the unification of OSF terms presented in this paper – which is currently being implemented – is relevant not only from a knowledge representation standpoint – as it provides more flexibility by allowing to model imprecise knowledge – but also to our current research towards the definition of similarity-based unification of OSF terms. Indeed, given both a subsumption and a similarity relation on \mathcal{S} , we are considering a method to define a fuzzy ordering on \mathcal{S} in order to reduce the similarity-based setting to the one of this paper. This will be the subject of a future publication.

Other research directions include the development of a fuzzy logic programming language (e.g., in the style of Login [2]) that supports a fuzzy subsumption relation and/or a similarity relation between sort symbols in order to provide approximate solutions to a program, or a fuzzy extension of the CEDAR Semantic Web reasoner [5].

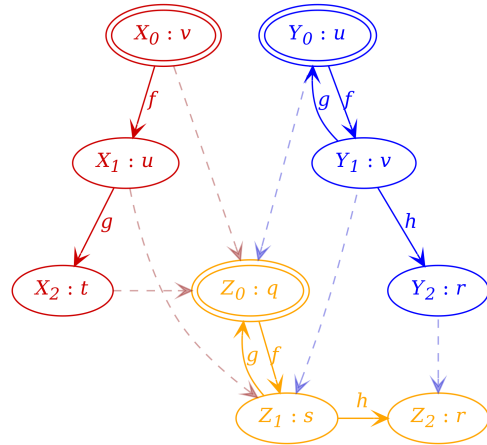


Figure 3: Fuzzy Unification of Example 3.7

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