Hybrid Linear Precoding Strategy for Multiuser Massive MIMO Millimeter Wave Systems

Baghdad Hadji, Lamya Fergani and Mustapha Djeddou

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Baghdad HADJI
Department of Telecommunication
University of Science and Technology
Houari Boumediene Algiers, Algeria
hadjb.ebm@gmail.com

Lamya FERGANI
Department of Telecommunication
University of Science and Technology
Houari Boumediene Algiers, Algeria
lamifer@msn.com

Mustapha DJEDDOU
Communication System Laboratory
Military Polytechnic School
BP 17 Bordj El Bahri, Algiers, Algeria.
djeddou.mustapha@gmail.com

***Abstract*—Multiuser-MIMO Millimeter wave based communication system has been considered as a current area of research for the future communication systems to serve multi-user simultaneously with a multiple data streams. Hybrid analog/digital architecture is the most satisfied architecture of the mmWave MIMO communication objectives and can be applied at both the base and mobile stations. Thus, exploiting the hybrid precoding/combining techniques offered by this architecture allow the realization of the multi-stream/multi-user transmission. We propose in this paper, a low computational complexity linear hybrid precoding schemes based on classical linear precoding and SVD decompostion of each channel user to avoid information exchange by the feedback where each user is powered by hybrid architecture. The simulation results demonstrate that our proposed two-stage hybrid linear precoding schemes can achieve a higher spectral efficiency compared with existing methods where each user is equipped with a single combiner and confirms its better management of the multi-user interference.**

***Index Terms*—Millimeter wave multi-user MIMO , Massive MIMO, hybrid precoding, MRT, ZF, MMSE**

I. INTRODUCTION

MmWave wireless communication is an enabling technology that has myriad applications to existing and emerging wireless networking deployments [1], as the 5th generation (5G) of wireless communication, Internet of Things (IoT) and vehicular communications. That is because, it can achieve multi-gigabit data rates benefiting from its abundant frequency resource [2] and helps applications that require weak latency [3]. On the other hand, propagation aspects are unique at mmWave due to the very small wavelength compared to the size of most of the objects in the environment [4]. Hence, communications in the millimeter wave band suffers from increased path loss exponents, higher shadow fading, blockage and penetration losses, etc., than sub-6 GHz systems leading to a poorer link margin than legacy systems [5]. However, achieving high quality communication links in mmWave systems requires employing large antenna arrays at both the access point or base station (BS) and the mobile stations (MS's) [6]. Interestingly, multiple-input multiple-output (MIMO) is one of the promising techniques, which can exploit large-scale antenna elements at transceivers to achieve high beamforming gains and combat huge attenuation and penetration loss at mmWave frequencies [7]. Fortunately, the shorter wavelength of the mmWave frequencies enables more antenna components to be packed in the same physical dimension, which allows for large scale spatial multiplexing and highly directional beamforming [8]. The communication systems based on massive MIMO architectures can be used precoding and combining principles as a protocol to exchange the data streams between its transmit and receive terminals. In conventional lower frequency systems, this precoding is usually performed in the digital baseband to have a better control over the entries of the precoding matrix [9]. Unfortunately, the high cost and power consumption of mixed signal components make fully digital baseband precoding unlikely with current semiconductor technologies [10]. Hybrid precoding, splitting MIMO processing into the analog and digital domains, has been proposed as an interesting approach to overcome this limitation and reduce the number of converters and RF (radio frequency) chains to be used [11]. Furthermore, hybrid architectures are one approach that can provide an opportunity to meet the multi-stream mmWave communication. In addition, the work in [10] showed that hybrid precoding/combining is capable of achieving almost the same performance of the fully digital design. In the current research interests of future communication systems, serve multi-user simultaneously with a multiple data streams becomes one of the dominant target.

A number of precoding methods for multi-user mmWave systems were proposed [12], [13]. In [12], phased-ZF algorithm is applied at the BS with hybrid beamforming, an analog RF beamforming constructed by inverting the phase of conjugated transposed vector of each channel user's, and then makes ZF precoding at baseband processing to reduce the interference between the users. The authors in [13] propose the beamspace MU-MIMO techniques to improve the performance of downlink multi-user multi-beam transmission. It is worth mentioning that this techniques increase the hardware complexity and power consumption in beam management. In [6], [14], [15], a two-stage hybrid precoding algorithm was proposed based on the different approach to design the hybrid procedure at the BS and analogue combiners at users side. While the analog combiners are decided at each MS, it is unable to perform the spatial multiplexing for high data rate transmission.

In this work, we develop a low computational complexity linear hybrid precoding schemes at BS and multi-user to enable multi-data stream in order to achieve high data rates due to a large amount of spatial degrees of freedom available in hybrid analog/digital architecture.

The rest of the paper is arranged as follows. In Section 2, we introduce the system and mmWave channel models, and we explain the constraint of multi-user hybrid precoding/combing design problem. The proposed low-complexity solution is presented in Section 3. Simulation results are conducted in Section 4. Finally, concluding remarks are drawn in Section 5.

We use the following notation: $\mathbf{A}$ is a matrix, $\mathbf{a}$ denotes a vector, a indicates a scalar, and $\mathcal{A}$ is a set.$\|\mathbf{A}\|_F$ stands for the Frobenius norm of $\mathbf{A}$, whereas $\mathbf{A}^T$,$\mathbf{A}^*$,$\mathbf{A}^{-1}$,are its transpose, Hermitian, and inverse, respectively. $\mathbb{E}[.]$ denotes expectation.

II. SYSTEM MODEL

In this work, we consider a multi-user millimeter wave MIMO system which consists of one base station (BS) and U users in a single cell, where the proposed architecture of all users and base station is fully hybrid to enable multi-stream data in dowlink communication as illustrated in Fig.1. BS is equipped with N_{BS} transmit antennas and N_{RF} RF chains to communicate with U mobile stations (MS), the u^{th} MS has N_{MS} receive antennas and N_{RF}^{MS} RF chain.

As we know in hybrid precoding scheme, the BS first processes the data streams digitally at the baseband using digital precoders, and then up-converts the digitally processed signals to the carrier frequency through RF chains, followed by an analog precoder which is implemented by analog phase shifters. Let $\mathbf{F}_{BB_u} \in \mathbb{C}^{N_{RF}\times N_S}$ and $\mathbf{F}_{RF} \in \mathbb{C}^{N_{BS}\times N_{RF}}$ denote the digital precoder, and analog precoder of the BS, respectively. The sampled transmitted signal is therefore

$$\mathbf{x} = \mathbf{F}_{RF}\sum_{u=1}^{U}\mathbf{F}_{BB_u}\mathbf{s}_u \tag{1}$$

Where $\mathbf{s}_u \in \mathbb{C}^{N_S\times 1}$ is u^{th} vector of transmitted symbols, and the received signal at user u is given by

$$\mathbf{r}_u = \mathbf{H}_u\mathbf{x} + \mathbf{n}_u \tag{2}$$

where $\mathbf{H}_u$ denotes the channel between the BS and user u, and $\mathbf{n}_u$ denotes the additive white Gaussian noise (AWGN) with zero mean and variance σ^2.

Since mmWave channels show limited scattering, we use a geometric channel model with Lu scatterers for the u^{th} mobile station. the channel $\mathbf{H}_u$ is expressed as

$$\mathbf{H}_u = \sqrt{\frac{N_{BS}N_{MS}}{L_u}}\sum_{i=1}^{L_u}\alpha_{ui}\mathrm{a}_{MS}(\theta_{ui})\mathrm{a}_{BS}^H(\phi_{ui}) \tag{3}$$

Where α_{ui} is the complex gain of the i^{th} path between user u and BS, it can define the channel type (Rayleigh, Rician or Nakagami). The variables ϕ_{ui} and $\theta_{ui} \in [0, 2\pi]$ are the i^{th} path's azimuth angles of departure or arrival (AoDs/AoAs) of the BS and i^{th} users, respectively. The steering vector in ULA configuration can be written as :

$$\mathrm{a}_{\mathrm{BS}}(\phi_{ui}) = \frac{1}{\sqrt{N_{\mathrm{tx}}}}\left[1, e^{j\frac{2\pi}{\lambda}d\sin(\phi_{ui})}, \ldots, e^{j(N_{\mathrm{BS}}-1)\frac{2\pi}{\lambda}d\sin(\phi_{ui})}\right]^T \tag{4}$$

Where λ denotes the wavelength of the signal, and d is the distance between antenna elements. The array response vectors at each users, $\mathrm{a}_{MS}(\theta_{ui})$, can be written in a similar fashion. At the receiver side, the u^{th} MS uses an analog combiner $\mathbf{W}_{RF_u} \in \mathbb{C}^{N_{MS}\times N_{RF}^{MS}}$ and then down-converts the signals to the baseband through RF chains, followed by a digital combiner $\mathbf{W}_{BB_u} \in \mathbb{C}^{N_{RF}^{MS}\times N_S}$ to process the received signal $\mathbf{r}_u$ as :

$$\begin{aligned}\mathbf{Y}_u &= \mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{H}_u\mathbf{F}_{RF}\mathbf{F}_{BB_u}\mathbf{s}_u \\ &+ \sum_{j\neq u}^{U}\mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{H}_u\mathbf{F}_{RF}\mathbf{F}_{BB_j}\mathbf{s}_j + \mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{n}_u\end{aligned} \tag{5}$$

Assuming transmitted symbols are gaussian vectors, such that $\mathbb{E}\left[ss^H\right] = \frac{P}{U}I_{N_S}$, the achievable rate of user u is given by [8]

$$\begin{aligned}\mathbf{R}_u = \log_2\Bigg(\Bigg|I_{N_S} + &\frac{\frac{P}{U}\mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{H}_u\mathbf{F}_{RF}\mathbf{F}_{BB_u}}{\frac{P}{U}(\sum_{j\neq u}^{U}\mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{H}_u\mathbf{F}_{RF}\mathbf{F}_{BB_j}} \\ &\times\frac{\mathbf{F}_{BB_u}^H\mathbf{F}_{RF}^H\mathbf{H}_u^H\mathbf{W}_{RF_u}\mathbf{W}_{BB_u}}{\mathbf{F}_{BB_j}^H\mathbf{F}_{RF}^H\mathbf{H}_u^H\mathbf{W}_{RF_u}\mathbf{W}_{BB_u} + R_n)}\Bigg|\Bigg)\end{aligned} \tag{6}$$

Where $R_n = \sigma^2\mathbf{W}_{BB_u}^H\mathbf{W}_{RF_u}^H\mathbf{W}_{RF_u}\mathbf{W}_{BB_u}$ is the noise covariance matrix after combining and P is the average total transmitted power.

In other hand, the sum-rate of the system is then $R = \sum_{u=1}^{U} R_u$. Therefore, the design of hybrid precoders and combiners can be achieved by formulating a joint optimization problem to maximize the sum-rate as

$$\begin{aligned}\left\{\mathbf{F}_{RF}^{opt},\left\{\mathbf{F}_{BB_u}^{opt}\right\}_{u=1}^{U},\left\{\mathbf{W}_{RF_u}^{opt}\right\}_{u=1}^{U},\left\{\mathbf{W}_{BB_u}^{opt}\right\}_{u=1}^{U}\right\} &= \operatorname{argmax}\sum_{u=1}^{U}\mathbf{R}_u \\ \textbf{s.t.}\quad \sum_{u=1}^{U}\|\mathbf{F}_{RF}\mathbf{F}_{BB_u}\|_F^2 &\leqslant P \\ \mathbf{F}_{RF} \in \mathcal{F}, \mathbf{W}_{RF_u} &\in \mathcal{W}\end{aligned} \tag{7}$$

Where the sets $\mathcal{F}$ and $\mathcal{W}$ are the RF beamforming codebook and RF combining codebook at each user, respectively. According to hybrid architecture, these sets are implemented using phase shifters which impose the constraints on the RF hardware, such as the availability of only quantized angles for the RF phase shifters. Furthermore, the digital precoder/combiner $\mathbf{F}_{BB_u}^{opt}$,$\mathbf{W}_{BB_u}^{opt}$need to be jointly designed with the analog beamforming/combining matrices.

In general, the optimization of (7) is a nonconvex problem due to the presence of the multiplication of the variables. Thus, obtaining the globally optimal solution of of this optimization problem is not only highly complex, but also intractable for practical implementation. Fortunately, by taking advantage of the knowledge of channel state information (CSI) in prior, we

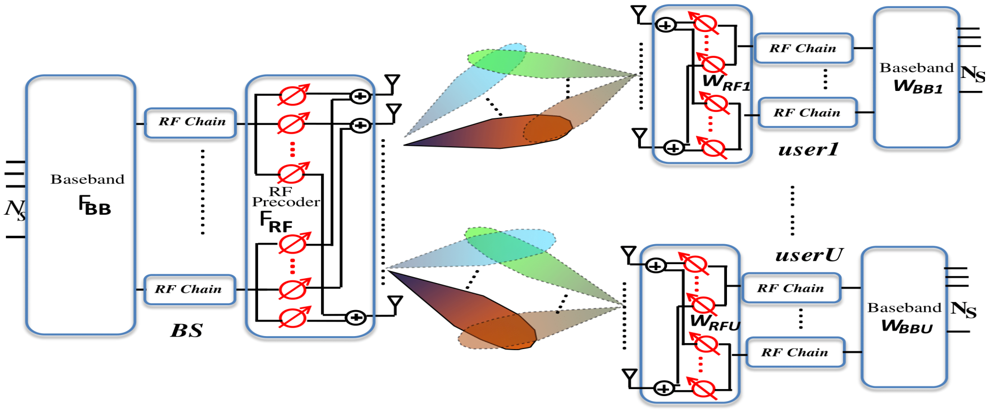

Fig. 1. MmWave multi-user system with hybrid precoding and hybrid combining.

exploit this CSI to design the hybrid precoders and combiners based on the two-stage strategy.

III. Two-Stage Hybrid Precoding

In this work, we assume that the user channel matrix $\mathbf{H}_u$ is known perfectly and instantaneously to both the transmitter and receivers and is well-conditioned to transmit N_S data streams such that rank$(\mathbf{H}_u) \geqslant N_S$. For each MS u, we denote the SVD of the channel matrix $\mathbf{H}_u$ by

$$\mathbf{H}_u = \mathbf{U}_u \Sigma_u \mathbf{V}_u^H \tag{8}$$

Where, $\mathbf{U}_u \in \mathbb{C}^{N_{MS} \times N_{MS}}$ and $\mathbf{V}_u \in \mathbb{C}^{N_{BS} \times N_{BS}}$ are the left singular matrix and the right singular matrix, respectively; $\Sigma_u \in \mathbb{C}^{N_{MS} \times N_{BS}}$ is diagonal matrix with singular values on its diagonal. As known, the SVD involves high computational complexity and is difficult to implement in hardware. To overcome this complexity, we define the SVD decomposition on two partitions, in which the left and the right singular matrix consist of singular vectors corresponding to the strongest singular value as:

$$\begin{aligned}\mathbf{H}_u &= \mathbf{U}_u \Sigma_u \mathbf{V}_u^H \\ &= \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix} \begin{bmatrix}\Sigma_1 & 0 \\ 0 & \Sigma_2\end{bmatrix} \begin{bmatrix}\mathbf{V}_1^H \\ \mathbf{V}_2^H\end{bmatrix} \\ &= \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^H + \mathbf{U}_2 \Sigma_2 \mathbf{V}_2^H\end{aligned} \tag{9}$$

Where, $\mathbf{U}_1 \in \mathbb{C}^{N_{MS} \times N_{RF}^{MS}}$ and $\mathbf{V}_1 \in \mathbb{C}^{N_{RF} \times N_{BS}}$ are semi-unitary matrices containing the left columns of unitary matrices $\mathbf{U}_u \in \mathbb{C}^{N_{MS} \times N_{MS}}$ and $\mathbf{V}_u \in \mathbb{C}^{N_{BS} \times N_{BS}}$, whereas Σ_1 is a diagonal matrix containing the largest singular values of $\mathbf{H}_u$ in decreasing order. Without loss of generality and for simplicity, we neglect the inter-user interference, then the received signal at each user can be expressed as

$$\mathbf{Y}_u = \mathbf{W}_{BB_u}^H \mathbf{W}_{RF_u}^H \mathbf{H}_u \mathbf{F}_{RF} \mathbf{F}_{BB_u} \mathbf{s}_u + \mathbf{W}_{BB_u}^H \mathbf{W}_{RF_u}^H \mathbf{n}_u \tag{10}$$

$$= \mathbf{W}_{BB_u}^H \mathbf{W}_{RF_u}^H \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^H \mathbf{F}_{RF} \mathbf{F}_{BB_u} \mathbf{s}_u + \mathbf{W}_{BB_u}^H \mathbf{W}_{RF_u}^H \mathbf{n}_u$$

To avoid further information exchange feedback and to manage the multi-user interference, we propose the design of the combiners and precoders separately, which is based on a low computational complexity hybrid linear precoding schemes, such as maximum-ratio transmission (MRT) and zero-forcing (ZF), and minimum mean square error (MMSE) scheme.

In hybrid structure, for analog RF precoding matrix, the MRT precoding, ZF precoding and MMSE precoding could be directly used to design $\mathbf{F}_{RF}$. Therefore, the corresponding analog precoding $\mathbf{F}_{RFmrt}$, $\mathbf{F}_{RFzf}$, and $\mathbf{F}_{RFmmse}$ are given by the following way:

$$\begin{cases}\mathbf{F}_{RFmrt} = \mathbf{V}_1 \\ \mathbf{F}_{RFzf} = \mathbf{V}_1 (\mathbf{V}_1 \mathbf{V}_1^H)^{-1} \\ \mathbf{F}_{RFmmse} = (\mathbf{V}_1^H \mathbf{V}_1 + \frac{U\sigma^2}{P} I)^{(-1)} \mathbf{V}_1^H\end{cases} \tag{11}$$

To finalize the first stage, we normalize the **RF** precoding as

$$\begin{cases}\mathbf{F}_{RFmrt} = \frac{1}{N_{BS}} \frac{\mathbf{F}_{RFmrt}}{\left\| \mathbf{F}_{RFmrt} \mathbf{F}_{RFmrt}^H \right\|_F} \\ \mathbf{F}_{RFzf} = \frac{1}{N_{BS}} \frac{\mathbf{F}_{RFzf}}{\left\| \mathbf{F}_{RFzf} \mathbf{F}_{RFzf}^H \right\|_F} \\ \mathbf{F}_{RFmmse} = \frac{1}{N_{BS}} \frac{\mathbf{F}_{RFmmse}}{\left\| \mathbf{F}_{RFmmse} \mathbf{F}_{RFmmse}^H \right\|_F}\end{cases} \tag{12}$$

The effective channel can be found by the product of the right singular matrix and each analog precoding obtained by (12). Therefore, the different effective channel is defined as follows

$$\begin{cases}\mathbf{H}_{eq1mrt} = \mathbf{V}_1^H \mathbf{F}_{RFmrt} \\ \mathbf{H}_{eq1zf} = \mathbf{V}_1^H \mathbf{F}_{zf} \\ \mathbf{H}_{eq1mmse} = \mathbf{V}_1^H \mathbf{F}_{RFmmse}\end{cases} \tag{13}$$

For the second stage, we use the low computational linear precoding strategy using each corresponding effective channel to design the digital precoding that is expressed by

$$\begin{cases}\mathbf{F}_{BBu_{mrt}} = \mathbf{H}_{eq1mrt}^H \\ \mathbf{F}_{BBu_{zf}} = \mathbf{H}_{eq1zf}^H (\mathbf{H}_{eq1zf} \mathbf{H}_{eq1zf}^H)^{(-1)} \\ \mathbf{F}_{BBu_{mmse}} = (\mathbf{H}_{eq1mmse} \mathbf{H}_{eq1mmse}^H + \frac{U\sigma^2}{P} I)^{(-1)} \mathbf{H}_{eq1}^H\end{cases} \tag{14}$$

At each MS side, a similar way can be applied to design a digital/analog combiner matrix using the linear precoding strategy and left singular matrix $\mathbf{U}_1$.

IV. Simulation Results

This section presents the simulation results to evaluate the performance of our proposed two-stage hybrid linear precoding schemes. We consider the multi-user system illustrated in Fig. 1 whith a BS equibed with $N_{BS} = 128$ antennas and $N_{RF} = 32$ RF chains and associated with 4 MSs, each having a $N_{MS} = 16$ antennas and $N_{RF}^{MS} = 8$ RF chains. Thus, we assume a ULA antenna configuration with the antenna spacing $d = \lambda/2$ at both the BS and each MS. The downlink channel for each user has $L = 3$ paths and Rayleigh distribute, the AoA and AoD are uniformly distributed in $[0, 2\pi]$. The mmWave MIMO system operates at 28 GHz with 100MHz bandwidth.

Figure 2 shows the average spectral efficiency performance of the proposed two-stage hybrid linear precoding schemes (including hybrid combiner/precoder MRT, hybrid combiner/precoder ZF and hybrid combiner/precoder MMSE) to perform multi-data stream communication.

We compared in figure 2 our proposed schemes with the perfect case using only Σ_1 to derive the optimal unconstrained precoding/combining communication, and the multi-user precoding proposed in [6] and [16]. The method proposed in [6] used an analog-only beamsteering combiner at each user and a multiple RF chains at the BS, i.e., one analog combining at each MS, which means that a single combiner only supports a single stream transmission. Whereas, the precoding proposed in [16] is based on beamspace MIMO (B-MIMO) scheme.

From figure 2, we can observe that the proposed hybrid linear precoding schemes achieves a high spectral efficiency and outperforms the results obtained by [6] and [16], because the efficiency of the system increases with the number of data stream.

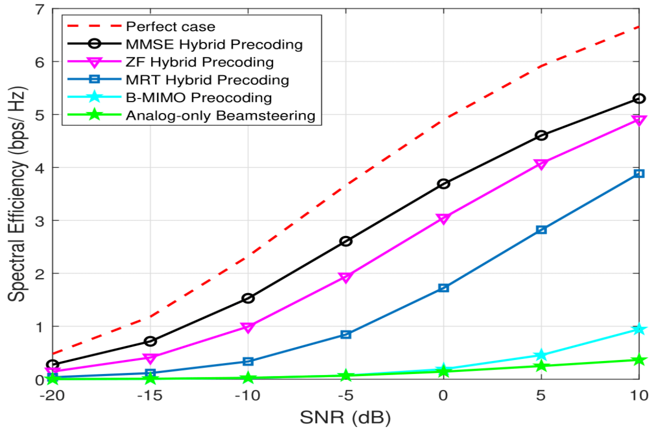

Fig. 2. Spectral efficiency v.s. SNR, achieved by different precoding methods.

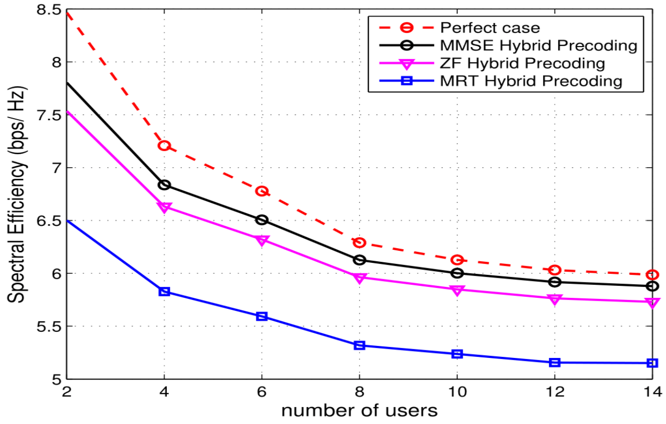

Fig. 3. Achievable Rate varying versus number of users U with SNR = 10dB

Fig. 3 compares the averaged achievable rates by varying the number of users for a fixed SNR = 10 dB in a multi-path scenario ($L_u = 3$) based on the proposed two-stage hybrid linear precoding schemes. We found that the hybrid combiner/precoder MRT decrease its performance for higher number of users, whereas, the proposed hybrid combiner/precoder ZF and hybrid combiner/precoder MMSE schemes show good outcomes with a similar trend, close to the perfect case.

V. CONCLUSION

In this paper, a low computational complexity hybrid linear precoding schemes are proposed for downlink mmWave MU-MIMO systems based on classical linear precoding and SVD decompostion of each user channel, where the BS and users are based on the hybrid analog/digital architecture to meet the multi-data stream in order to increase the efficiency of the system. The proposed schemes applied to both BS and multi-users avoid the information exchange feedback. The simulation results showed that our proposed two-stage hybrid linear precoding schemes can achieve a higher spectral efficiency compared with the methods proposed in [6] and [16], and confirms the better management of the multi-user interference offered by our proposed schemes. Accordingly, our proposed hybrid linear precoding schemes achieves high data rates due to a large amount of spatial degrees of freedom available in massive MIMO systems.

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