Nonlinear Event-Triggered Networked Feedback Control System Under Data-Rate Constrains

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Abstract— **The paper describes an Event-Triggered Nonlinear Control (ETNC) approach for Networked Feedback Control System (NFCS) under data-rate constraints. The nonlinear approach is based on the predefined sliding variable defined by the system states with a nonlinear switching function, which ensures the system stability by holding the variable in the prescribed boundary. The stability boundary of the sliding variable is subject to the preselected triggering condition, which selection is a tradeoff of system performance and network data-rate constraints. The primary purpose of triggering conditions is lowering network resources utilization while ensuring the proper performance of the NFCS. Regarding network constraints, the minimum inter-event time of controller update is derived. The efficiency of the proposed NFCS method is confirmed with the results on the real system.**

Keywords—**Nonlinear system, event-triggering, networked control system, sliding mode control**

I. INTRODUCTION

Sliding mode control (SMC) is an effective approach to ensure the prescribed performance of a closed-loop system, despite external disturbances and system uncertainty [1],[2]. Depending on the controller structure, the sliding mode controller is straightforward to implement and requires much computational time. All controllers in the real-time system are implemented in a discrete form, which results in a hybrid system where the continuous and discrete systems are interconnected [3]. The most commonly used approach for controller implementation is a sample and hold technique or time triggering approach (TT). Time triggering means that the controller output is updated at equidistant time intervals, also known as a sampling time. Such TT closed-loop system is more suitable to design due to the vast of the developed techniques and approaches for time sampled systems. On the other hand, the TT system requires constant resources utilization and data transmissions over the network system.

Event-Triggering (ET) approach of a closed-loop system offers an alternative to the TT [4]. Regarding the TT in the ET system, the closed-loop system is updated based on the trigger rule evaluation. In other words, the controller is updated when the system states violate the triggering rule, which means that the controller is no longer updated periodically with fixed time intervals. Such an implementation of the controller is more efficient than the TT implementation and requires fewer computational resources, especially when the sliding manifold is reached. Regarding the latter, ET is beneficial for the networked control system (NCS), where the trigger mechanism reduces network transmission and is suitable for systems with data-rate constraints [5],[6]. The network constraints with variable Round Trip Time (RTT), limited data transmission, and package drops are insufficient for the NCS[7] . Mentioned network parameters considerably reduce system performances and can lead to unstable operation. The presented work introduced an SMC controller design with an associated triggering rule, which ensures NCS stability and takes all the network parameters into account during the design procedure. The derived event-triggered sliding mode controller (ET-SMC) introduces triggering boundaries regarding the admissible lower inter-event time value and network delay [8]. The ET-SMC for NCS is divided into two steps. The first step introduces an SMC controller design with preselected system dynamics and parametrized sliding variables [9],[10]. The second step involves triggering boundary selection regarding the system tracking performance and NCS uncertainty robustness. In comparison to the similar linear ET paradigms, the presented approach still ensure SMC properties and effectively lowers the computational burden and network usage.

The controller parameter selection can be presented as an optimization procedure. The optimal parameter selection can be evaluated as a tradeoff between network utilization regarding NCS uncertainties and closed-loop performance, such as tracking capability, transient performance, network delay, etc. The assessment of the admissible lower inter-event time of the ET shows the direct influence of the ET-controller on the network utilization during the reaching and sliding phase of the sliding variable evolution. The efficiency of the proposed controller is evaluated on the real-time system.

II. SLIDING MODE CONTROLLER DESIGN

For the sliding mode controller (SMC) synthesis the given system is used,

$$\dot{x}_1 = x_2 \tag{1}$$

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$$\dot{x}_2 = -bx_2 + gv + d,$$

where $x(t) = [x_1(t)\ x_2(t)]^T \in \mathbb{R}^2$ is a state vector and $v(t) \in \mathbb{R}$ is the control variable. The parameters $g:\mathbb{R} \to \mathbb{R}$ and $b:\mathbb{R} \to \mathbb{R}$ are system parameters where $d:\mathbb{R} \to \mathbb{R}$ is a disturbance. For SMC design, the boundary of system parameters are given, $0 < b < b_{\max}$, $g_{\min} < g < g_{\max}$, $[g_{\min}, g_{\max}, b_{\max}] \in \mathbb{R}_{>0/\infty}$. For system tracking capability, new system states are introduced, $\xi_1 = x_d - x_1$, $\xi_2 = \dot{x}_d - x_2$, where x_d is desired value with its derivative $\dot{x}_d$. The transformed system is given as,

$$\begin{aligned}\dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -b\xi_2 - gv + \tilde{d},\end{aligned} \tag{2}$$

where $\xi = [\xi_1, \xi_2]^T$, $\tilde{d} = -d + \ddot{x}_d + b\dot{x}_d$ and holds $\sup_{t\geq 0} |\tilde{d}(t)| \leq \Delta_d < \infty$. The sliding variable is designed as $s = c\xi$ for $c \in \mathbb{R}^2$, where is $c = [c_1\ \ 1]$, $c_1 > 0$. Differentiating of $s = c\xi$ with respect to time gives,

$$\begin{aligned}\dot{s} &= \dot{\xi}_2 + c_1\dot{\xi}_1 = \\ &= (c_1 - b)\xi_2 - gv + \tilde{d} < 0.\end{aligned} \tag{3}$$

Regarding (3), $\Delta_d < \infty$ and sliding property, which brings the sliding variable to the sliding manifold, $s, \dot{s} = 0$ the SMC controller can select as,

$$v = g^{-1}\left((c_1 - b)\xi_2 + \rho\, sign(s)\right), \tag{4}$$

where holds $\rho > \Delta_d$. After the SMC controller design (4), the ET mechanism will be introduced in the next section. The controller (4) contains a nonlinear term, the solution of the feedback system (2),(3) with controller (4) is understood in the Filippov sense [8].

III. Event-Triggered sliding mode control for NCS

The event-triggered rule derivation is based on the analysis of the reaching phase stability of the sliding variable [9]. It is worthy of mentioning that the discrete implementation of SMC can not completely reach a sliding manifold. As a result, the quasi-sliding mode is obtained [8], [11], where the sliding variable is limited with boundary $|s| \leq \Omega$, $\Omega \in \mathbb{R}_+/\mathbb{R}_\infty$, where Ω it is subject to the sampling time, sliding parameter, and disturbance Δ_d. Furthermore, the presented work is limited to the ET approach, where the band Ω will be determined regarding the trigger mechanism and preselected inter-event time. The ET-SMC after two consecutive updates is given as,

$$v(t) = g^{-1}\left((c_1 - b)\xi_2(t_n) + \rho\, sign(s(t_n))\right), \tag{5}$$

where t_n is the last update, t is the current time between two updates, and is $t \in [t_n, t_{n+1})$.

Theorem 1: *Consider system (2) with the sliding manifold $s = 0$. The parameter β is given such that,*

$$\|(c_1 - b)\|\|e_2(t)\| < \beta, \tag{6}$$

for all $t > 0$, where is $e_2(t) = \xi_2(t) - \xi_2(t_n)$. The event triggering is established if the controller gain is selected as,

$$\rho > \beta + \Delta_d. \tag{7}$$

Proof: Before continuing to prove, remaining ET error variables are introduced, $e_1(t) = \xi_1(t) - \xi_1(t_n)$, and $e(t) = \xi(t) - \xi(t_n)$. For the stability test, the Lyapunov function is presented $V(t) = \frac{1}{2}s(t)^2$ for the time interval $t \in [t_n, t_{n+1})$, where $n \in \mathbb{Z}_{\geq 0}$. Differentiation V with respect to time t the derivative $\dot{V}$ is given as,

$$\dot{V} = s\dot{s} = s\left((c_1 - b)\xi_2 - gv + \tilde{d}\right). \tag{8}$$

Substituting the controller (5) in (8) it gives,

$$\begin{aligned}\dot{V} &= s\left((c_1 - b)\xi_2(t) - gv(t_n) + \tilde{d}(t)\right) \\ &= s\left((c_1 - b)\xi_2(t) - (c_1 - b)\xi_2(t_n) - \rho\, sign(s(t_n)) + \tilde{d}(t)\right) \\ &= s\left((c_1 - b)(\xi_2(t) - \xi_2(t_n)) - \rho\, sign(s(t_n)) + \tilde{d}(t)\right) \\ &\leq s(c_1 - b)\underbrace{(\xi_2(t) - \xi_2(t_n))}_{e_2(t)} - |s|\rho + |s|\Delta_d \\ &\leq s(c_1 - b)e_2(t) - |s|\rho + |s|\Delta_d \\ &\leq |s|\beta - |s|\rho + |s|\Delta_d \\ &\leq -|s|(\rho - \beta - \Delta_d) \\ &\leq -\psi|s|,\end{aligned}$$

where is $\psi > 0$. Concerning the condition (7) and assumption $sign(s(t_n)) = sign(s(t))$, it is to be noted that the sliding variable is approaching the sliding manifold, where is $s = 0$. The above is true if at the time of triggering $t = t_n$ holds $e_2(t_n) = e_2(t) = 0$, then the sliding variable s is bounded with Ω, where is,

$$\begin{aligned}|s(t) - s(t_n)| &= |c\xi(t) - c\xi(t_n)| = \|c\|\|e\| \\ &\leq \|c\|\|k\|\|e_2\| \\ &< k\frac{\|c\|}{\|(c_1 - b)\|}\beta = \tilde{k}\beta,\end{aligned} \tag{9}$$

regarding $\|e_2\| \leq \|e\|$ and $k\|e_2\| = \|e\|$. The parameter k is defined as $k = \sqrt{1 + \frac{\|(c_1 - b)\|^2\alpha^2}{\beta^2}}$ and α is an upper limit of the $\sup_{t\geq 0}|e_1(t)| \leq \alpha < \infty$. The boundary Ω is defined as $\Omega = \{\xi \in \mathbb{R}, |s| = |c\xi| < \tilde{k}\beta\}$, where the triggering rule in (6) can be defined as,

$$\|e_2(t)\| > \beta\|(c_1 - b)\|^{-1}, \tag{10}$$

which is the end of the proof.

After the stability analysis of the sliding variable with triggering condition, the stability of the remaining system in (2) with controller (5) needs to be assessed. Regarding the reaching phase boundary (9), it can be derived $\xi_2 = s - c_1\xi_1$, where holds $\dot{\xi}_1 = s - c_1\xi_1$. With the introduction of the Lyapunov function $V = \frac{1}{2}\xi_1^2$, the stability can be assessed as,

$$
\begin{aligned}
\dot{V} &= \xi_1 \dot{\xi}_1 \\
&= \xi_1 \left(s - c_1 \xi_1 \right) \\
&= -c_1 \left\| \xi_1 \right\| \left(\left\| \xi_1 \right\| - \frac{1}{c_1} \left\| s \right\| \right).
\end{aligned}
$$

With respect to conditions (6)(9), the system is stable if it holds $\|\xi_1\| - c_1^{-1}\|s\| > 0$. Thus, closed-loop system is stable with respect to s, and the system trajectory ξ_1 is bounded by,

$$
\|\xi_1\| < \frac{k\|c\|}{\|c_1\|\|(c_1 - b)\|}\beta. \tag{11}
$$

IV. Networked Control System Structure

The structure of the network control system is depicted in Fig 1. The controller algorithm is executed on the network computer, where the triggering rule is evaluated on the plant. We assume that the plant has a real-time system with computational ability and communication interfaces. The real-time system on the plant side is used for noncomplex computation such as triggering condition evaluation, signal conditioning, and communication capability. For the given ET-SMC implementation, the User Datagram Protocol (UDP) is used. The data have been transmitted over different networks hops, where additional time delay and package loss may occur. The package loss in the network is modeled as a loss delay [6], where the maximal allowed Round Trip Time (RTT) of the network is used for package loss detection. The plant side uses a dedicated package-loss timer, and if the watchdog timer is expired, then the request for new data from the server is demanded. For the package loss occurrence, we assume that two consecutive losses can not be accrued.

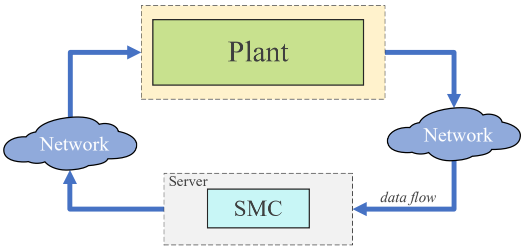

Fig. 1. Networked controller structure with ET-SMC.

The controller feedback structure is presented in Fig. 2, where the triggering condition determines the network usage. The controller (5) is implemented on the server, and the triggering mechanism is on the plant side.

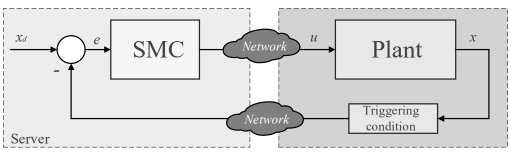

Fig. 2. ET-SMC feedback configuration.

The inter-event time of the ET-SMC is determined regarding the error analysis of the two consecutive sampled states:

$$
\frac{d}{dt}\|e(t)\| \le \left\| \frac{d}{dt} e(t) \right\| = \left\| \frac{d}{dt} \begin{bmatrix} \xi_1(t) - \xi_1(t_n) \\ \xi_2(t) - \xi_2(t_n) \end{bmatrix} \right\| = \left\| \frac{d}{dt} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} \right\|, \tag{12}
$$

where is $\xi(t_n) = 0$, according to the last update. Substitute (12) in (2), (5) which gives,

$$
\begin{aligned}
\frac{d}{dt}\|e(t)\| &\le \left\| \begin{bmatrix} 0 & 1 \\ 0 & -b \end{bmatrix} \xi(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tilde{d}(t) - \begin{bmatrix} 0 \\ g \end{bmatrix} v(t_n) \right\|, \\
&= \left\| \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -c_1 \end{bmatrix}}_{A_c} \xi(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{B_d} \tilde{d}(t) - \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_c} \rho\, sign(s(t_n)) \right\| \\
&= \left\| A_c \left(e(t) + \xi(t_n) \right) - B_c \rho\, sign(s(t_n)) + B_d \tilde{d}(t) \right\| \\
&\le \|A_c\|\|e(t)\| + \|A_c\|\|\xi(t_n)\| + \|B_c\|\rho + \|B_d\|\Delta_d.
\end{aligned}
$$

The solution of the differential equations is,

$$
\|e(t)\| \le \frac{\|A_c\|\|\xi(t_n)\| + \|B_c\|\rho + \|B_d\|\Delta_d}{\|A_c\|} \left(e^{\|A_c\|(t - t_n)} - 1 \right), \tag{13}
$$

where the minimal inter-event time $\tau = t - t_n$ is determined as,

$$
\tau_{\min} \ge \frac{1}{\|A_c\|} \ln \left(\frac{k\beta\|A_c\|}{\|(c_1 - b)\|\left(\|A_c\|\|\xi(t_n)\| + \|B_c\|\rho + \|B_d\|\Delta_d\right)} + 1 \right). \tag{14}
$$

It can be seen that the inter-event time depends on triggering conditions β and selected controller parameters c_1 and ρ. Regarding the uncertainty of the network, the delay η_n is introduced with the update time t_n. The update sequence $\{t_n + \eta_n\}_{n=0}^{\infty}$ corresponds to the update time t_n and means that the controller is not updated with the last states, wherein the inter-event time is extended by delay value η_n. Hence the error (13) grow till the next update time t_{n+1}. The triggering sequence is admissible with respect to $\{\eta_n\}_{n=0}^{\infty}$, if $t_{n+1} > t_n + \eta_n$, $n \in \mathbb{Z}_{\ge 0}$ and the triggering rule (6), (10) ensure system stability. The derivation of the delay boundary, where the triggering rule ensures the system stability, is similar to the derivation of the inter-event time in (13), (14). For given derivation, we assumed that the controller (5) at the time $t \in [t_n, t_n + \eta_n)$ is not updated with the current state $\xi(t_n)$, whereby the further updates are executed at $t \in [t_n + \eta_n, t_{n+1} + \eta_{n+1})$, and the analysis involves the controller structure with past value $v(t) = g^{-1}\left((c_1 - b)\xi_2(t_{n-1}) + \rho\, sign(s(t_{n-1}))\right)$.

The admissible inter-event time is caused by the delay, which ensures system stability with triggering condition (10) is,

$$
\tau_n = \frac{1}{\|A_c\|} \ln \left(\frac{k\beta\|A_c\|}{\|(c_1 - b)\|\left(\|A_c\|(\|\xi(t_n)\| + \|\xi(t_{n-1})\|) + \|B_c\|\rho + \|B_d\|\Delta_d\right)} + 1 \right). \tag{15}
$$

The system is stable, and the boundary (11) is preserved if it holds $\eta_n \le \tau_n$. For proper controller parameters selection is necessary to assume the maximally allowed delay in the network. The delay boundary is given as, $\sup_{n \ge 0} |\eta_n| \le \Delta_\eta < \infty$.

V. Experimental Results

The parameters of the system presented in (1), (2) are, $b = 3.3$, $g = 0.897$, $\Delta_d = 7.1$. The triggering rule and UDP servises are implemented on the ARM® Cortex®-M7 based STM32F7xx MCU with Digital-Signal Processing and Floating-Point Unit (DSP and FPU) and operating frequency of 216MHz. The

UDP communication is implemented based on the LwIP open-source TCP/IP stack with 100BASE-TX-Ethernet. The SMC controller is implemented on the computer with Matlab-script and UDP communication package. The ET-SMC controller parameters are selected regarding the measured network RTT and package loss. Each communication thread contains 120 bytes, where the real-time system ID, RTT values, and state measurements are transferred over the network. The measured RTT outside of the subnet is approximated to 6.7 ms, where the fastest time is 3.5 ms, and the slowest is 8.5 ms. Regarding the RTT measurement, the watchdog timer is set to 13ms and $\tau_{\min} \geq 7\,ms$. To ensure the proper performance of the feedback system, the max delay value is selected as $\Delta_\eta = 13\,ms$, where controller parameters regarding (14), (15) are, $c_1 = 12.4$, $\rho = 22.5$, $\beta = 20$ and $\beta = 11.5$. The real-time results are presented in Figs. 3-5.

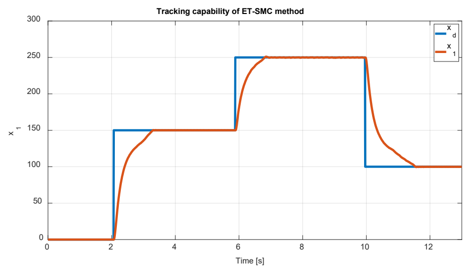

Fig. 3. Networked ET-SMC angle control.

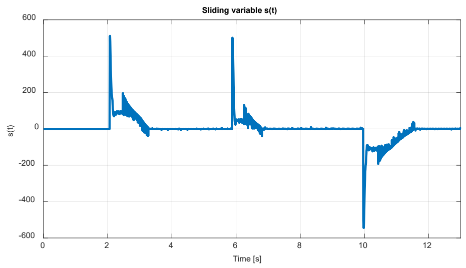

Fig. 4. ET-SMC sliding variable.

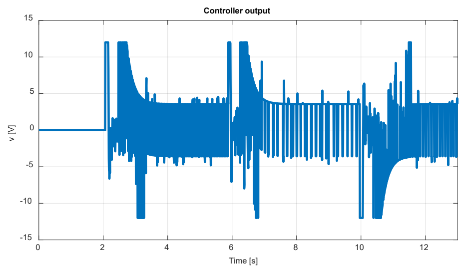

Fig. 5. ET-SMC controller output.

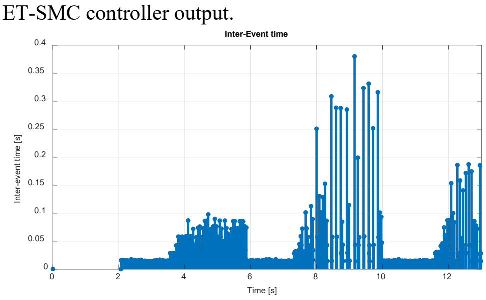

Fig. 6. ET-SMC inter-event time.

The figures present the tracking capability of angle control x_1 , the evolution of the sliding variable, controller output, and inter-event time values. The results show that the ET implementation is reliable and ensures the system's proper performance, with lower network usage regarding Fig. 6. The minimum inter-event time in the transient response is 16.2ms, which confirms the adequate selection of the controller parameters regarding the network limitation and system dynamic. The inter-event time in the stationary phase is extended to approx. 400 ms, which indicates the relaxation of the network usage and system resources. Regarding the TT method, the ET approach is beneficial in the NCS systems, where the network constraints are crucial.

VI. CONCLUSION

The paper presents the event-triggering nonlinear controller implementation for the networked control system. The approach is comparable to the time triggering execution and is beneficial for the NCS system with data rate constraints, where the network constraints can be considered during the controller design. The work is a good research starting point matter for multi-agent, distributed control, and task scheduling in embedded systems. The central supervised server system can share its computation capacity with other distributed systems and remotely control multiple sub-plants, where the relaxation of frequency of network requests can be significantly lowered and pre-estimated.

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