Characterization of Model Uncertainty Features Relevant to Model Predictive Control of Lateral Vehicle Dynamics

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***Abstract*—The information about a system's dynamics represented by measurement data sets are often confined to regions of restricted operations where the system is not sufficiently excited for model identification purposes. Experiments performed in closed-loop with safety constraints allow only for reduced order modeling. In the paper, a set of low order models are identified from real experimental data of the lateral dynamics of an electric passenger car. Low order models are advantageous for on-line computation in model-based control, though uncertainty due to neglected dynamics may deteriorate control performance and constraint satisfaction. The effect of uncertainty is analyzed by controller cross-validation where a controller designed based on one model is evaluated on other models playing the role of the true system. This method allows us to qualify not only model-controller pairs, but to determine the properties of input data and model uncertainty, which lead to more useful data sets, more robust and better performing controllers than the others.**

***Index Terms*—uncertainty modeling, model predictive control, feature selection, classification**

I. Introduction

Model predictive control (MPC) recently became an appealing technique even for fast nonlinear systems with state and input constraints due to the real-time feasibility of problem specific optimization algorithms [1], [2]. One of the main issues regarding MPC is robustness, the ability of maintaining feasibility, stability and performance specifications in the course of on-line computations in the presence of model uncertainty and noise [3].

In this paper we restrict the questions about robustness to the *analysis* of control performance. Specifically, we are interested in finding the properties of the model and its uncertainty that mostly influence the accuracy of a nominal reference tracking model predictive controller. Data-based control oriented model identification and uncertainty modeling have a great literature both in the prediction error and the deterministic robust control frameworks. In the most efficient methods, the criteria of modeling and control are closely related. Therefore, the windsurfer approach [4], the unfalsified control approach [5] and other iterative schemes [6], [7], developed for classical and modern robust control, are not directly applicable for MPC. Data driven MPC oriented modeling is presented in [8], where model and controller are optimized iteratively using closed-loop data collected on the true plant. Instead of applying a robust MPC algorithm, the predictor model is selected to provide the best closed-loop performance. In contrast to the above works that develop a model directly applicable for control, our goal is to perform a preliminary analysis to reveal the model's important properties on which any identification for control method should focus in order to improve closed-loop MPC performance.

The structure of the paper is organized according to the following summary of the proposed method.

- Given a large amount of open- or closed-loop segmented data, for each segment several low order output-error *(OE) models* are identified. One obtained model may be valid only in a small region of operation (Section II).
- Then closed-loop simulations are performed, where every model is considered as a predictor model of an *MPC* reference tracking problem, and every other model is considered one after the other as the plant. The result is a large matrix of tracking performance values (Section III).

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- We choose an initial set of features that we think it might be relevant and physically meaningful for every pair of model and plant. These features should relate to the region of operation where the model or plant is valid, the model's and plant's dynamics, and the properties of the reference signal to be tracked. The next task is to *select the features* with the greatest influence in deciding whether a pair of model and plant have acceptable performance or not (Section IV).
- Given a predictor model, a second goal is to provide *bounds on the tolerable model uncertainty* based on the selected features. The contribution of this method is to present hints for experiment design and identification on what are the relevant properties of a good model for MPC (Section V).

Conclusions are presented in Section VI.

II. Set of Local Models

Data for identification purposes are collected on an electric vehicle that is equipped with actuators supporting autonomous functions. Closed-loop steering experiments are conducted where a PID controller ensures keeping the road and additional pseudo random binary noise (PRBS) disturbance on the control signal excites the lateral dynamics for identification. The input of the plant is steering angle, δ the output and scheduling variable are yaw rate, r and speed v, respectively. The experiments cover a range of speed $[0, 30]/3.6\frac{m}{s}$ and steering angle of $[-30, 30]\frac{\pi}{180}$ radians.

In order to obtain linear models data segments of *constant speed* are selected. The segments represent transient motion in steering for ensuring sufficiently exciting input for identification (for example, lane change maneuvers, or increasing/decreasing the radius of cornering). Then for each data segment multiple OE models are identified of orders $n = 1, 2, 3, 4$ in the form

$$r_p(t) = \frac{b_{p,1}q^{-1} + \ldots + b_{p,n}q^{-n}}{1 - (a_{p,1}q^{-1} + \ldots + a_{p,n}q^{-n})}\delta_p(t) + \nu_p(t), \quad (1)$$

where t denotes discrete time $t = 1, \ldots, N_p$, q denotes the forward time shift operator, $a_{p,i}, b_{p,i}$, $i = 1, \ldots, n$, are constant coefficients determining the rational transfer function $W_p(q)$ of the model, and $\nu_p(t)$ is the simulation error. All models identified from different data segments and different orders are indexed and collected in set $\mathcal{P} \triangleq \{W_p | p = 1, \ldots, P\}$. Model W_p is identified from IO data δ_p and r_p. The constant speed of segment p is denoted by v_p.

III. Reference Tracking Predictive Control with Constraints

Assume that each model is stable and represents the vehicle's behavior, of certain quality depending on the model order, under the conditions of its experiment has been conducted. This implies that under the same conditions, the true plant is locally approximated by these models, i.e., the models may play the role of the plant in certain closed-loop reference tracking simulations, where the reference signal $r_{ref,p}$ to be followed is generated by the model and the corresponding input used in the identification, i.e.,

$$r_{ref,p}(t) \triangleq W_p(q)\delta_p(t), \quad p = 1, \ldots, P. \quad (2)$$

So, on one hand, every identified OE model plays the role of the plant in simulations where $r_{ref,p}$ is to be tracked.

On the other hand, we have to choose models from $\mathcal{P}$ that will play the predictor in the MPC problem. To keep the computational complexity as low as possible we are interested only in the first order models with $n = 1$. Let $\mathcal{M} \triangleq \{W_m | m = 1, \ldots, M, W_m \text{ is of order } 1\} \subset \mathcal{P}$ denote the set of tested predictors.

Remark. *An alternative to the choice of reference signal* (2) *could be the measured yaw-rate of the true plant used in the identification. The difference between the two signals is the residual in the identification problem, which is the smallest possible for the optimal OE model given the model order. Since model uncertainty defined in Section IV is expressed in terms of identified OE models reference signal* (2) *is preferable because it can be accurately tracked when the plant and predictor models are the same, thus zero uncertainty model will guarantee good control performance.*

Let's simulate all model-plant pairs $(W_m, W_p) \in \mathcal{M} \times \mathcal{P}$ in the following MPC problem.

$$\min_{u_0,\ldots,u_{H-1}} V_{m,p}(u, t) \quad (3)$$

$$s.t. \quad u_k \in [\underline{u}, \bar{u}] \quad (4)$$

$$u_k - u_{k-1} \in [\underline{\Delta u}, \overline{\Delta u}] \quad (5)$$

where the horizon length determined by data segment p is denoted by H, (4) and (5) define control input constraints. The quadratic criterion is defined on the horizon by

$$V_{m,p}(u, t) \triangleq \sum_{k=0}^{H-1} Q(r_m(t + k|t) - r_{ref,p}(t + k))^2 + \sum_{k=0}^{H-1} R(u(k) - u(k-1))^2 \quad (6)$$

where Q and R are parameters, $r_m(t + k|t)$ is the predicted output of model W_m with input $u(k)$, and initial state $r_{m,p}(t - 1)$ "measured" at time $t - 1$ on plant W_p. Some on-line and explicit MPC solutions for the above problem are presented and compared in terms of computational time and memory requirement in [9]. After solving the convex quadratic program, input $\delta_{m,p}(t) := u_0$ is applied to plant W_p, and as the simulation evolves the horizon is shifted forward in time to finally obtain the performance of the pair (W_m, W_p)

$$J_{m,p} \triangleq \frac{1}{N_p - H - n} \sum_{t=n+1}^{N_p - H} (r_{m,p}(t) - r_{ref,p}(t))^2. \quad (7)$$

measuring the mean squared reference tracking error of the simulation. The final result of the simulations is the $M \times P$ performance matrix J with elements $J_{m,p}$. If $m = p$ and $R = 0$ the solution of the optimization problems give back δ_p

and $J_{p,p} = 0$. An $R > 0$ is required, however, to ensure some degree of robustness.

IV. Feature Selection

The first of our questions is: which properties of the model-plant pair lead to good tracking performance? Certainly, $J_{m,p}$ depends directly on both models and the reference signal, and indirectly, on the dynamics of the true plant and the identification inputs. The tool we have chosen to answer this question is to build a binary classification model $\mathcal{C}$ that maps property variables (features) into two classes. One class denoted by C_1 should contain the model-plant pairs that result in tolerable closed-loop performance, i.e., with $J_{m,p} < c$. The other class C_0 is the complement set. During training the classifier, weights for the features are developed. At the end, very small weight of a feature will imply its insignificance.

In order to minimize the bias of the classifier model, initially all features that may influence the tracking performance should be included in the feature set. Let

$$x_{m,p}^T \triangleq [\bar{v}_m,\ |\bar{\delta}_m|,\ b_1,\ a_1,\ \bar{v}_p - \bar{v}_m,\ |\bar{\delta}_p| - |\bar{\delta}_m|,\ A_{m,p}(\omega_1),\ \ldots,\ A_{m,p}(\omega_s),\ \phi_{m,p}(\omega_1),\ \ldots,\ \phi_{m,p}(\omega_s),\ D_p(\omega_1),\ \ldots,\ D_p(\omega_s)] \quad (8)$$

denote the row vector of features of one model-plant pair where $\bar{v}_m = \frac{1}{N_m}\sum_{t=1}^{N_m} v_m(t)$ and $|\bar{\delta}_m| = \left|\frac{1}{N_m}\sum_{t=1}^{N_m}\delta_m(t)\right|$ characterize the region of operation where the identification data of model W_m were collected; b_1, a_1 are the model parameters completely determining its dynamics. The next two features are the differences between the operating regions of the model and the plant. The model uncertainty (or "neglected dynamics") is defined in the frequency-domain in a multiplicative form by $W_p(e^{j\omega}) = (1 + \Delta_{m,p}(e^{j\omega}))W_m(e^{j\omega})$. $A_{m,p}$ and $\phi_{m,p}$ are the magnitude and phase of $\Delta_{m,p}$ at the chosen frequency grid. Finally, $D_p(\omega_i)$ are the samples of Welch's (smoothed) power spectral density (PSD) estimate for input δ_p at the specified frequencies. Input δ_p characterizes both the true measured output r_p and the reference signal $r_{ref,p}$, thus it's PSD influences both the quality of model W_p and the hardness of tracking the reference signal in the control problem.

All features $x_{m,p}^T \in \mathbb{R}^{n_f}$ of all model-plant pairs are stacked into a $MP \times n_f$ feature matrix X. Similarly, for all pairs the corresponding performance values are stacked into the MP long column vector Y. By fixing the level c of acceptable performance the vector of class labels is derived as $C = (Y < c)$ (i.e., $C(i)$ is logical 1 if the ith pair is acceptable). The data set X, C is divided into training and test sets, and for feature selection for classification neighborhood component analysis (NCA) is applied [10], [11].

Table I shows the trained classifiers confusion matrices for the training and the test data sets, respectively. The number of good model-plant pairs is the sum of the last row, none is classified incorrectly from the training set and 156 ($\approx 8.3\%$) is misclassified from the test set. The first row shows that the amount of good pairs that are estimated as bad pairs is less than 1%. Fig. 1 shows that most of the incorrectly classified pairs has a tracking performance close to the given acceptance level $c = 0.4$.

TABLE I
Confusion Matrices of Binary Classification

True Classes	Estimated Classes			
	Training Data Set		Test Set	
	Bad Pairs	Good Pairs	Bad Pairs	Good Pairs
Bad Pairs	14528	3	14390	142
Good Pairs	0	1883	156	1726

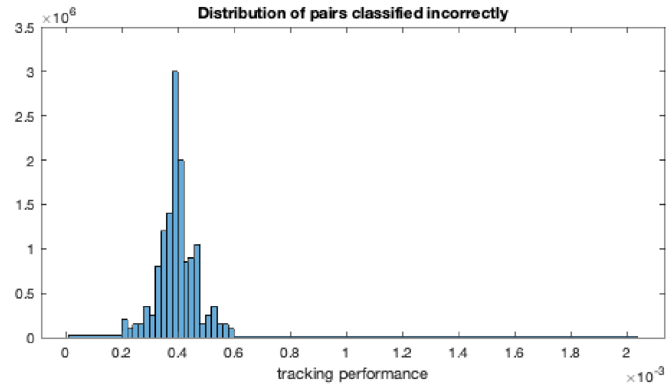

Fig. 1. Histogram of the performance values of the incorrectly classified pairs.

The weights of each features are plotted in Fig. 2. Features with very small weights are not necessary for decision making. It can be concluded from the list of significant features, that performance depends

- on the region of operation represented by the model features 1 and 2;
- on the dynamics of the model (or the plant) beyond model uncertainty (features 3, 4);
- uncertainty magnitude and phase of the multiplicative uncertainty at some frequencies (features 7,. . ., 26).

We note that some poles (complex pole pairs) of the plants are in the frequency range of $\omega \in [10, 20]$ rad/s where both magnitude and phase uncertainties are important.

Knowing the important features support any further design tasks, like experiment design, model identification and uncertainty modeling.

V. Uncertainty Modeling

The goal of this section is to construct model uncertainty bounds for possible further robust control design. Since features 1,...,4 are important in the classification, the uncertainty bounds may depend on these features. The following analysis is working on a restricted set of models which are very similar in the first four features. Then, within this group, zero uncertainty must imply good performance by the classifier model. Let $\mathcal{M}_1 \subset \mathcal{M}$ denote this subset of models. The corresponding set of 595 pairs are characterized by $\bar{v}_m \in [4.08,\ 4.23]$m/s, $\bar{\delta}_m \in [2.5,\ 4.3]$deg, $b_1 \in [0.43,\ 0.47]$ and

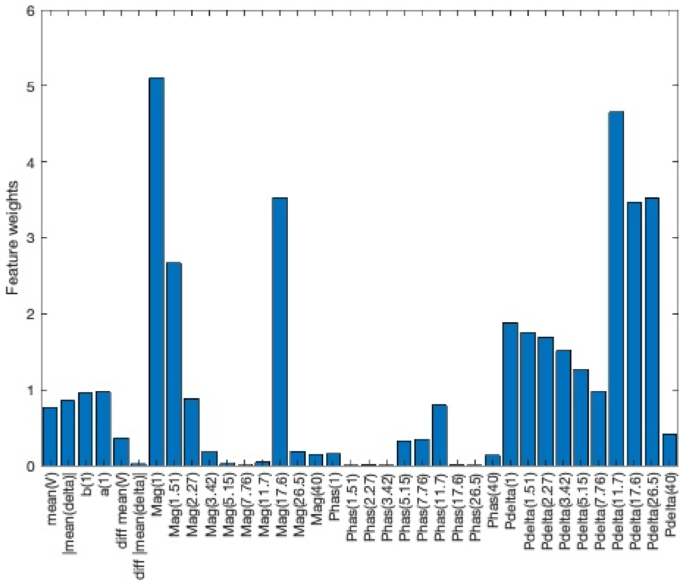

Fig. 2. Feature weights showing at least 9 insignificant features.

TABLE II
UNCERTAINTY BOUNDS FOR A GROUP SIMILAR MODELS

	Minimum	**Maximum**
$\bar{v}_p - \bar{v}_m$	-0.42061 [m/s]	0.2205 [m/s]
$A_{m,p}(1)$		0.14319
$A_{m,p}(1.51)$		0.15672
$A_{m,p}(2.27)$		0.10167
$A_{m,p}(3.42)$		0.11318
$A_{m,p}(17.6)$		1.4981
$A_{m,p}(26.5)$		1.6293
$A_{m,p}(40)$		1.4167
$\phi_{m,p}(1)$	-4.4908 [deg]	6.0588 [deg]
$\phi_{m,p}(5.15)$	-3.5473 [deg]	6.1264 [deg]
$\phi_{m,p}(7.76)$	-5.7413 [deg]	5.5751 [deg]
$\phi_{m,p}(11.7)$	-5.8913 [deg]	3.5648 [deg]
$\phi_{m,p}(40)$	-6.8919 [deg]	6.6991 [deg]

$a_1 \in [0.68,\ 0.71]$. Within this group of pairs the goal is to find (soft) upper and lower bounds for the uncertainty $\Delta_{m,p}$

$$A_{m,n}(\omega_i) \leq \bar{A}(\omega_i), \tag{9}$$

$$\underline{\phi}(\omega_i) \leq \phi_{m,p}(\omega_i) \leq \bar{\phi}(\omega_i), \tag{10}$$

for all $i = 1, \dots, s$ and $(W_m, W_p) \in \mathcal{M}_1 \times \mathcal{P}$, such that

- all pairs satisfying all of the bound constraints should be classified as good performing pairs
- all pairs violating any of the bounds should be classified as bad performing pairs

One possible method is to minimize the number of incorrectly classified pairs in the bounds. Table II shows the obtained bounds which lead to 16+16 misclassifications out of the 595 pairs.

The obtained uncertainty bound can also be viewed as the region of validity of a given model. Performing the above procedure for multiple regions of operations and models (features 1,...,4), one could develop a set of models that together with their validity regions may cover the whole working region of the vehicle dynamics. But the elaboration and validation of this problem is out of the scope of the paper.

VI. CONCLUSIONS

From experimental data that covered a wide range of operating regions of a vehicle's lateral dynamics, a large set of low order local linear models were identified. Important features of models and model uncertainty that are relevant in contributing model predictive tracking control performance, were revealed with the help of closed-loop simulations and a binary classifier. The results provide important information for model selection, experiment design and identification. By using the classifier, frequency-domain bounds were derived for the amplitude and phase of multiplicative model uncertainty.

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