Turbulent Flow Modeling Using Lattice Boltzmann Method

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TURBULENT FLOW MODELING WITH LATTICE BOLTZMANN METHOD

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Abstract: **The Lattice Boltzmann method, a molecule Kinetic-based approach, is presented to solve fluid dynamics. Based on theory of turbulence and molecule kinetics, an extended Lattice Boltzmann equation is put forward to solve turbulent flow with high Reynolds number, in which turbulence sub-grid model is used to simulate vortex viscosity as well as turbulence relaxation time is introduced to modify the normal LBGK equation. The method to evaluate the turbulence relaxation time with particle distribution function is proposed combining with Smagorinsky turbulence model. Furthermore the extended Lattice Boltzmann method is applied to simulate the flows around square cylinder in the range of Reynolds number from Re=62 to Re= 9000.**

Keywords: Turbulent flow, extended LBM equation, High Reynolds number, square cylinder.

I. INTRODUCTION

Turbulence refers to the state of a fluid, liquid or gas, in which the velocity presents in every point a swirling character: vortices whose size, location and orientation vary constantly. Turbulent flows are therefore characterized by a very disordered appearance, a behavior that is difficult to predict and the existence of numerous spatial and temporal scales. Such flows occur when the kinetic energy source that sets the fluid in motion is relatively intense in front of the viscosity forces that the fluid opposes to move. Conversely, laminar is called the character of a regular flow.

This work concerns the study of turbulent flows around obstacles using the Lattice Boltzmann method (LBM).

As a first step, the lattice model D2Q9 is used to numerically model the flow of a fluid in a horizontal channel [3]. The model is validated by comparison of the analytical profiles is numerical speeds in the case of Poiseuille's flow. In a second step, the scheme is then applied to simulate the flow around a square-section cylinder, the modeling of turbulence is approached, a large-scale Reynolds number scale modeling algorithm is developed and the model of Smagorinsky is implemented in the code. Indeed a numerical test case is used to validate the code and the simulation of a turbulent flow is realized.

II. LATTICE BOLTZMANN MODEL

A. *Lattice Bolzmann Equation:*

First, The D2Q9 is a 2-dimensional model. The position of the particles is limited to the nodes of lattice. Momentum and microscopic speeds are considered equivalent because the mass is uniform (1 mass unit or mu), (lu) is the fundamental measure of the length in the model LBM, ts the time step is the unit of time [1]. Figure 1 shows the Cartesian network and speeds $\vec{e}_a \quad a \in \{0,1,2,3,4,5,6,7,8\}$. We have $\vec{e}_0 = \vec{0}$ because the particle remains at rest.

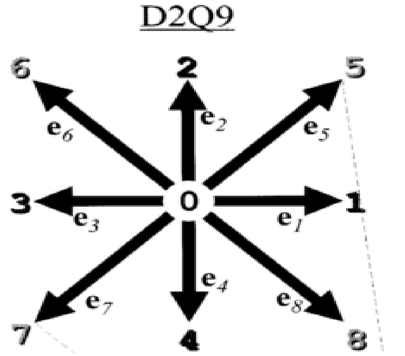


Figure 1: ***D2Q9 Lattice and velocities***

The velocity magnitude of e_1 through e_4 is $1 lu.ts^{-1}$, from e_5 to e_8 is $\sqrt{2} lu.ts^{-1}$.

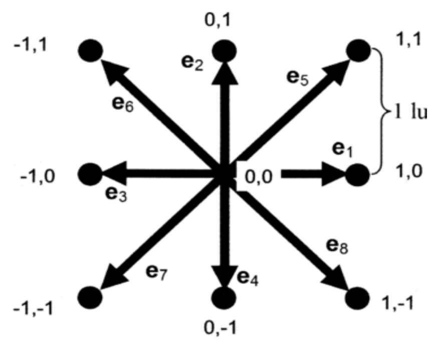


Figure 2: ***D2Q9 x,y velocity components***

The macroscopic fluid density is:

$$\rho = \sum_{a=0}^{a=8} f_a \quad (1.1)$$

The macroscopic average u is:

$$u = \frac{1}{\rho}\sum_{a=0}^{a=8} f_a e_a \quad (1.2)$$

BGK approximation:

$$\underbrace{f_a(x + e_a\Delta t, t + \Delta t) = f_a(x,t)}_{\text{Streaming}} - \underbrace{\frac{f_a(x,t) - f_a^{eq}(x,t)}{\tau}}_{\text{Collision}} \quad (1.3)$$

$f_a(x,t)$ is the density fuction.

τ is the relaxation time.

$$f_a^{eq}(x) = w_a \rho(x)\left[1 + \frac{3e_a u}{c^2} + \frac{9}{2}\frac{(e_a u)^2}{c^4} - \frac{3}{2}\frac{u^2}{c^2}\right] \quad (1.4)$$

Is the local equilibrium function.

$w_a = \frac{4}{9}$ if a=0; $w_a = \frac{1}{9}$ if $a \in \{1,2,3,4\}$;

$w_a = \frac{1}{36}$ if $a \in \{5,6,7,8\}$

B. *Extended Lattice boltzmann*

In general, turbulence is composed of large-scale flows and small-scale fluctuations the energy cascade of large scales flows towards small-scale fluctuations; here the turbulence theory is applied to calculate the turbulent viscosity of all scales of the vortex movement since the anisotropic scales and the swirl viscosity model are introduced at LBE. The large scales are simulated exactly by the distribution functions f and the equilibrium distribution functions f_{eq}, this resolved scale flux determines the local effective relaxation time to calculate the unresolved scales of the turbulent motion, hence the relaxation time tau (total) is computed locally for each cell in the simulation domain for each time step (ts), across sub-mesh scales.

Discrete LBM equation becomes [2]:

$$f_\alpha\left(x + e_\alpha \Delta t, t + \Delta t\right) = f_\alpha\left(x,t\right) - \frac{1}{\tau_{tot}}\left(f_\alpha - f_\alpha^{ea}\right) \quad (1.5)$$

$$\tau_{tot} = \tau_0 + \tau_t \quad (1.6)$$

τ_{tot} need to be determined describing the dynamics of turbulent fluctuations, following the relationship between the relaxation time and the viscosity of the flow, the total relaxation time is expressed by:

$$\tau_{tot} = 3\frac{\Delta t}{\Delta x^2}\left(\nu_0 + \nu_t\right) + \frac{1}{2} \quad (1.6)$$

The eddy viscosity is governed by the model of Smagorinsky:

$$\nu_t = \left(C_S \Delta\right)^2 \left|\bar{S}\right| \quad (1.7)$$

As C_S is the Smagorinsky constant, Δ is the filter size and $|\bar{S}|$ is the magnitude of the strain rate tensor.

$$\left|\bar{S}\right| = \sqrt{2S_{ij}S_{ij}} \quad (1.8)$$

$\bar{S}_{ij}$ can be evaluated with resolved-scale non equilibrium momentum tensor:

$$S_{ij} = -\frac{3}{2\rho\tau_{tot}\Delta t}\prod_{ij}^{(1)} \quad (1.8)$$

Where $\prod_{ij}^{(1)} = \sum_\alpha e_{\alpha i} e_{\alpha j}\left(f_\alpha - f_\alpha^{eq}\right)$ then the eddy viscosity is expressed as:

$$\nu_t = \frac{3}{\sqrt{2}\tau_{tot}\Delta t \rho}\left(C_S \Delta\right)^2 \sqrt{Q} \quad (1.9)$$

Where: $Q = \prod_{ij}\prod_{ij}$, consequently, the total relaxation time τ_{tot} can be solved by:

$$\tau_{tot} = \frac{1}{2}\left(\sqrt{\tau_0^2 + \frac{18}{\Delta x^2 \rho}\left(C_S \Delta\right)^2 \sqrt{2Q}} + \tau_0\right) \quad (1.10)$$

III. Simulation of Flow Around Square Cylinder

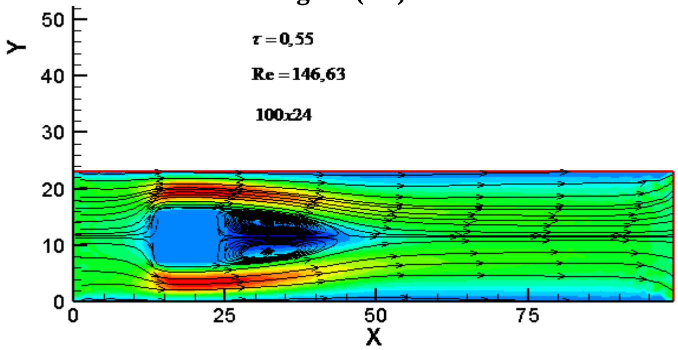
The code of simulation is LB2D [4], developed by FIU (Florida International University). This code has been used to simulate Poiseuille's flow and large eddy simulation.

A. *Poiseuille's flow:*

The lattice used for this simulation is 100x24, $u_{x-in} = u_{x-out}$ <0,1lu/ts.

The instabilities in the flow can be triggered at a low Reynolds number (Re) in the presence of a square stop obstacle of 12 lu positioned at the position (18lu, 12 lu). As **Re** increases from the slip flow the separation appears with vortex formation behind the obstacle. These vortices lengthen with decreasing number of Reynolds. Non-linear terms are currently in action.

Figure (3.a)



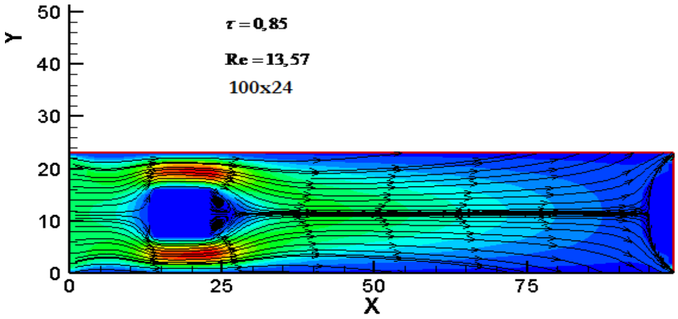


Figure (3.b)

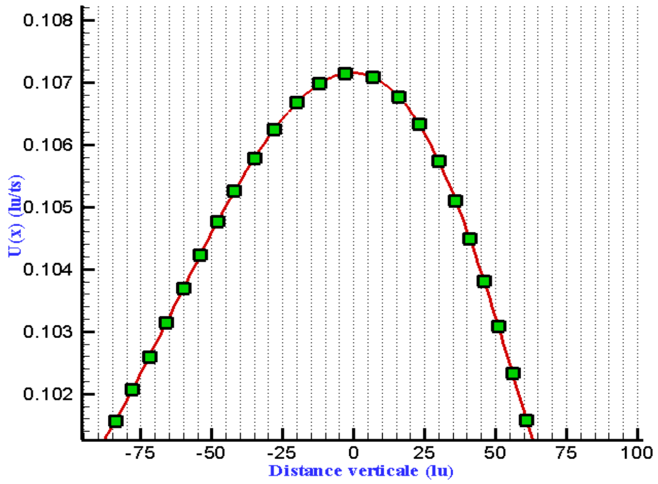


Figure 4

This figure shows the numerical profile of the speed in a section compared to the analytical profile. There is a total agreement between the numerical and analytical results.

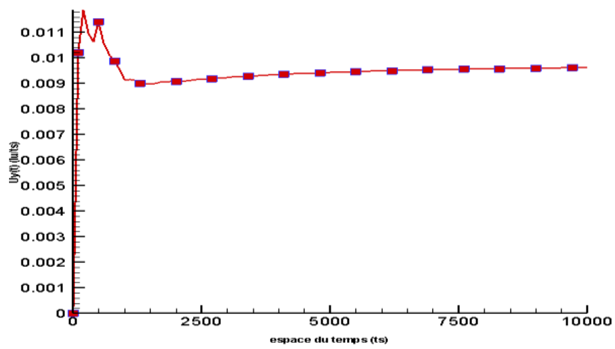


Figure (5.a) and (5.b)

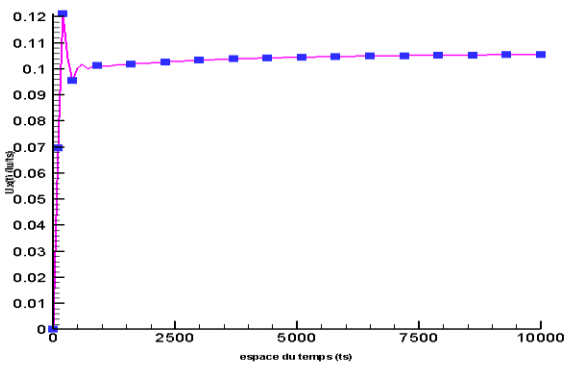


Figure 5 shows for laminar Reynolds flows the records of speeds u_x and u_y as a function of time are practically a straight line.

B. Simulation of turbulent flow:

After integrating the model of Smagorinsky we obtain these results, streamlines are obtained with tecplot:

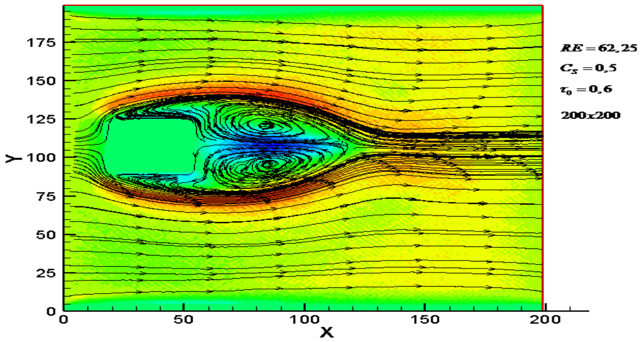


Figure (6.a)

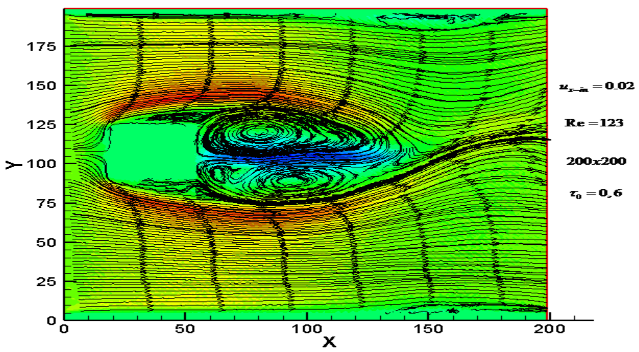


Figure (6.b)

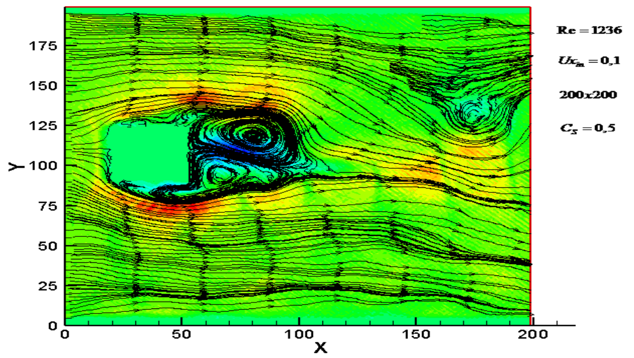


Figure (6.c)

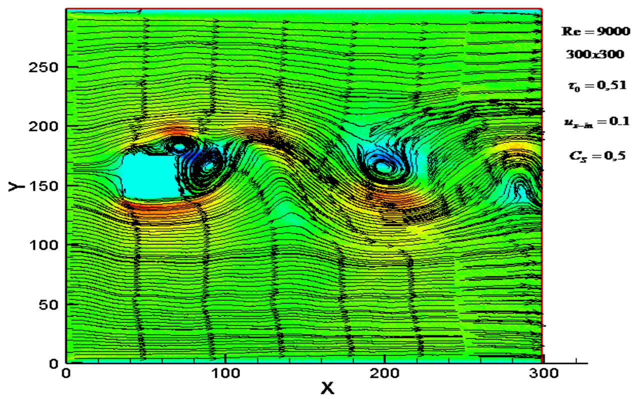


Figure (6.d)

On the Figure (6.a) The Lattice used is 200x200. Initial velocity $u_{x-in} = 0.01, \tau = 0,6$, The simulation performed using LBM assumes the periodic bounce-back boundary conditions on the walls of the flow line, and the total number of iterations is 10000. The visualization of the density, vorticity and velocity shows the formation of two vortices symmetrical to the axis of flow.

On the figure (6.b), by taking $u_{x-in} = 0.02$, we can see the appearance of the first asymmetric vortices of turbulence, with appearance of small vortices on the lateral surfaces of the cylinder with square section.

On the figure (6.c) we observe Detachment of vortices at Re = 1236 Scheme 200x200.

On the figure (6.d), we notice a strong detachment of the vortices to a very large number of Reynolds.

IV. CONCLUDING REMARKS

In this work, the linearized Boltzmann lattice equation is solved numerically. For the implementation of the Boltzmann lattice method, we opted for the simulation of the Poiseuille's flow to compare with the analytical solution. The solutions obtained show a very good agreement with the analytical results.

On the other hand, the flow around an obstacle is considered. We have determined the critical Reynolds numbers for which there is appearance of Von Karman vortices or appearance of dynamic instabilities corresponding to an asymmetry of these vortices.

To conclude this work, both the theoretical and the practical aspects of the Boltzmann lattice method have been studied. It has been shown that this method can be used to simulate delicate problems.

ACKNOWLEDGMENTS

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